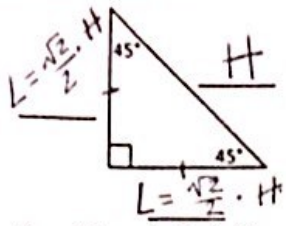
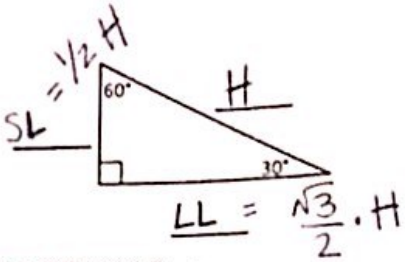
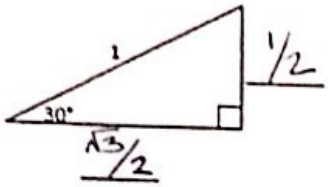
The Unit Circle

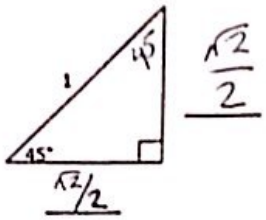
- We're going to be using special right triangles, so first, remind yourself of the patterns of 45-45-90 triangles and 30-60-90 triangles:

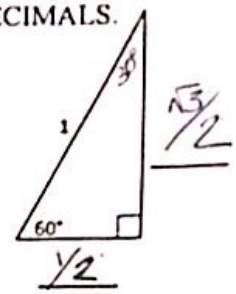




- Use special right triangles to fill in the lengths of the missing sides. NO DECIMALS.







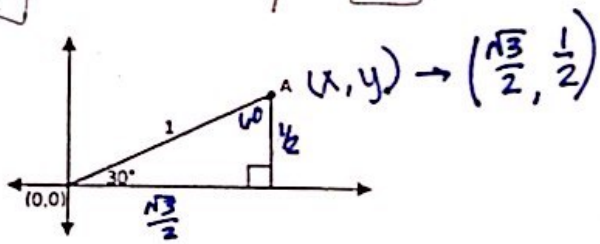
- Now use what you know about trigonometry to write the following sine and cosine ratios for the angles shown in these triangles. SIMPLIFY THOSE RATIOS!

sin 30° = $\frac{O}{H} = \frac{1/2}{1} = \boxed{\frac{1}{2}}$ sin 45° = $\frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$ sin 60° = $\frac{\frac{\sqrt{3}}{2}}{1} = \boxed{\frac{\sqrt{3}}{2}}$

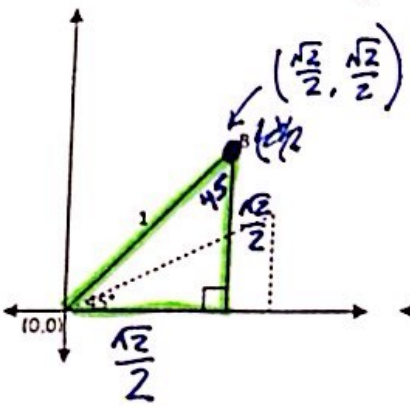
cos 30° = $\frac{\frac{\sqrt{3}}{2}}{1} = \boxed{\frac{\sqrt{3}}{2}}$ cos 45° = $\frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$ cos 60° = $\frac{\frac{1}{2}}{1} = \boxed{\frac{1}{2}}$

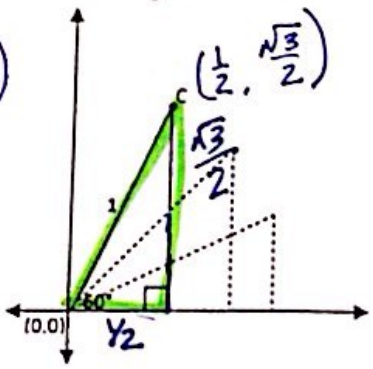
- Next we'll take one of those triangles and put it on a coordinate plane:

What would the *x*- and *y*- coordinates of point A (formed by a 30° angle) be? *(Hint: Use the lengths of the sides of the triangle.)*

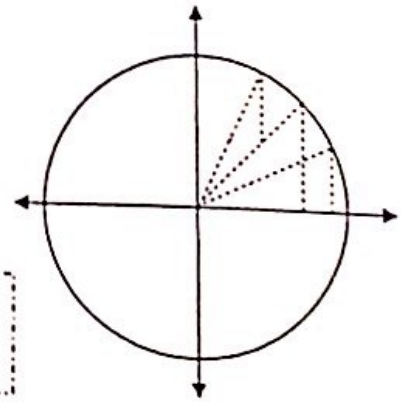


- Do the same for a 45° angle and a 60° angle:





If you look at the three points we have traced out so far, they all lie on a circle centered at the origin.



What is the radius of this circle?

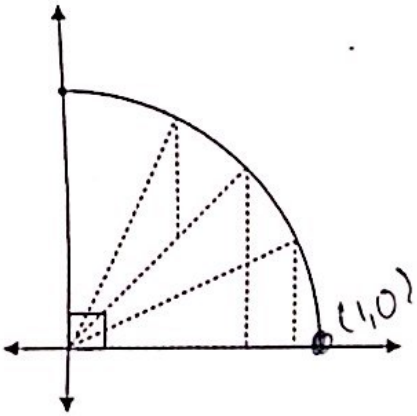
» Use the work you did with the triangles on the front page to fill in the following table:

Angle measure	Coordinate on unit circle	Value of sine	Value of cosine
30°	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

What relationship do you notice between the coordinate on the unit circle and the values of sine and cosine?

$\boxed{(\cos\theta, \sin\theta)}$

» Look at the point shown below, which corresponds to 90°.



What are the coordinates of this point? (0,1)

So what is the value of sin 90°? 1

What is the value of cos 90°? 0

What would be the coordinates of the point that corresponds to 0°? (1,0)

What is sine of 0°? 0

What is cosine of 0°? 1

» More practice: *Use the unit circle you just made - your answers should be exact, NO DECIMALS.*

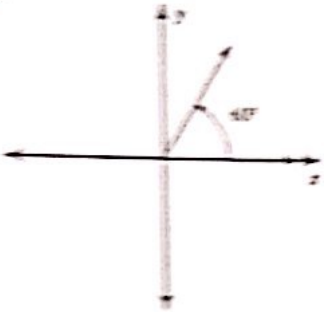
1. sin 120° =
2. cos 180° =
3. cos 135° =
4. sin 315° =
5. cos 240° =
6. sin 330° =

Exact Trig Values of Special Angles

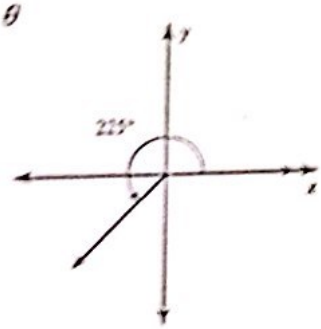
Date_____ Period____

Find the exact value of each trigonometric function.

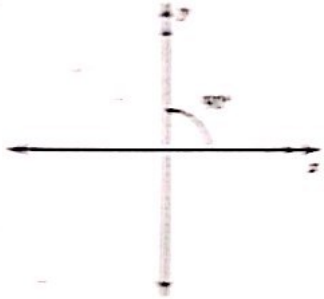
1) $\tan\theta$



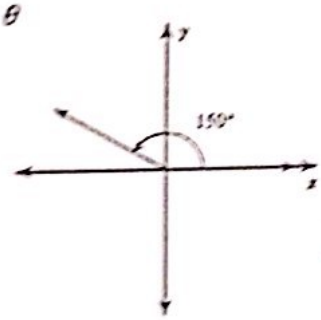
2) $\sin\theta$



3) $\sin\theta$

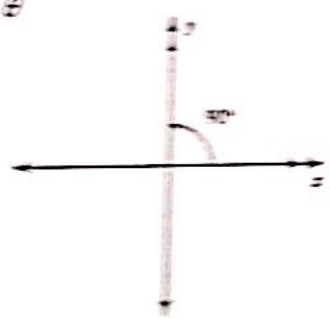


4) $\cos\theta$

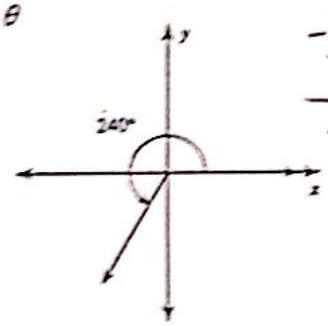


$\tan\theta = \frac{\sin\theta}{\cos\theta}$

5) $\cos\theta$



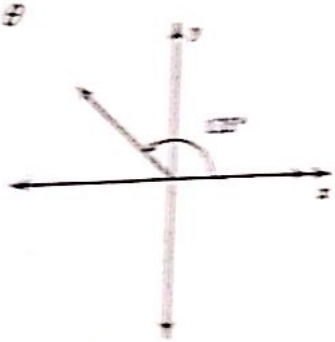
6) $\tan\theta$



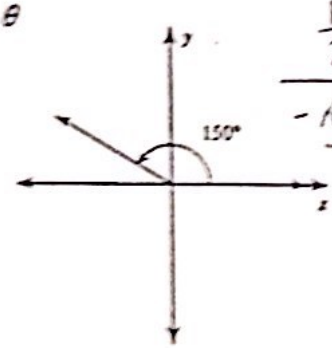
$$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{-\sqrt{3}}{\cancel{2}} \cdot \frac{-\cancel{2}}{1}$$

$$\boxed{=\sqrt{3}}$$

7) $\cos\theta$



8) $\tan\theta$

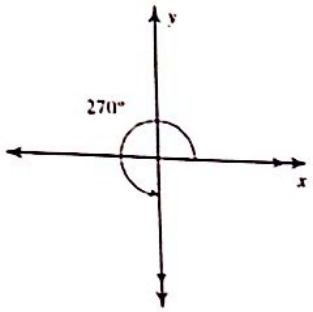


$$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{-\cancel{2}}{\sqrt{3}}$$

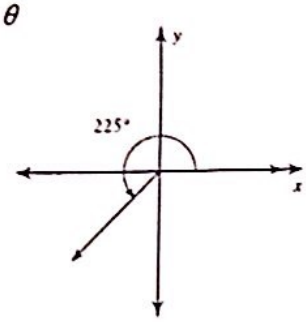
$$= \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} =$$

$$\boxed{\frac{-\sqrt{3}}{3}}$$

9) $\cos\theta$



10) $\tan\theta$



11) $\cos 270°$

12) $\sin 0$

13) $\cot\frac{7\pi}{4}$

14) $\csc\frac{2\pi}{3}$

$\frac{1}{\sin 225}$ ← 15) $\csc 225° = \frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\cancel{2}\sqrt{2}}{\cancel{2}}$

$= \boxed{-\sqrt{2}}$

16) $\sin 300°$

17) $\csc 90°$

18) $\tan 240°$

19) $\sin\frac{\pi}{4}$

20) $\tan 120°$

---

21) $\tan -\frac{13\pi}{6}$

22) $\cos -630°$

23) $\cos 990°$

24) $\csc -\frac{31\pi}{6}$

25) $\csc -\frac{5\pi}{6}$

26) $\cos -\frac{17\pi}{3}$

27) $\sin\frac{29\pi}{6}$

28) $\sec 945°$

29) $\cos -\frac{11\pi}{2}$

30) $\sin -2\pi$