

Solving Systems of Equations w/ Matrices

$$1. \frac{\begin{bmatrix} -15 & -5 & -5 \end{bmatrix}}{-5} = \frac{-5x}{-5}$$

$$\begin{bmatrix} 3 & 1 & 1 \end{bmatrix} = x$$

$$2. -2B - \begin{bmatrix} 3 \\ 1 \\ -11 \end{bmatrix} = \begin{bmatrix} -9 \\ 9 \\ 11 \end{bmatrix} \rightarrow \frac{-2B}{-2} = \begin{bmatrix} -6 \\ 8 \\ 0 \end{bmatrix} \div -2$$

$$+ \begin{bmatrix} 3 \\ 1 \\ -11 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ -11 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$$

$$3. \begin{bmatrix} 2 & 9 \\ -1 & -1 \end{bmatrix} x = \begin{bmatrix} 38 \\ -5 \end{bmatrix}$$

① Inverse: $\det = 7$
 $\text{inv} = \frac{1}{7} \begin{bmatrix} -1 & -9 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1/7 & -9/7 \\ 1/7 & 2/7 \end{bmatrix}$ ☺

② multiply on correct side to cancel! (left)

$$\begin{bmatrix} -1/7 & -9/7 \\ 1/7 & 2/7 \end{bmatrix} \begin{bmatrix} 2 & 9 \\ -1 & -1 \end{bmatrix} x = \begin{bmatrix} -1/7 & -9/7 \\ 1/7 & 2/7 \end{bmatrix} \begin{bmatrix} 38 \\ -5 \end{bmatrix}$$

* must multiply inverse on same side! (left)

$(2 \times 2) \quad (2 \times 1)$
 2×1

$$x = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$4. \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} A - \begin{bmatrix} -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 49 \\ 21 \end{bmatrix}$$

$$+ \begin{bmatrix} -3 \\ 7 \end{bmatrix} + \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} A = \begin{bmatrix} 46 \\ 28 \end{bmatrix}$$

A = $\begin{bmatrix} 9 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} 1/2 & -1/2 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} A = \begin{bmatrix} 1/2 & -1/2 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} 46 \\ 28 \end{bmatrix}$$

Inverse original

$$\textcircled{5} \quad \begin{aligned} -x + 5y &= 9 \\ -4x - 4y &= -12 \end{aligned}$$

→ Find all coefficients

$$\begin{bmatrix} -1 & 5 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -12 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 1 \quad 2 \times 1$

→ in calc:

$$A: \begin{bmatrix} -1 & 5 \\ -4 & -4 \end{bmatrix}$$

$$B: \begin{bmatrix} 9 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

so $x=1$ and $y=2$

$$\textcircled{6} \quad \begin{aligned} 2x - y - 2z &= -14 \\ x - 5y - 4z &= -16 \\ 4x + z &= -2 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & -2 \\ 1 & -5 & -4 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -14 \\ -16 \\ -2 \end{bmatrix}$$

→ in calc:

$$A: \begin{bmatrix} 2 & -1 & -2 \\ 1 & -5 & -4 \\ 4 & 0 & 1 \end{bmatrix}$$

$$B: \begin{bmatrix} -14 \\ -16 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 6 \end{bmatrix}$$

① word problems → you write the equations!
 could be 2, 3, 4... equations/variables
 # of equations = # of variables

→ write equations

$$5h + 6s = 54$$

$$4h + 12s = 72$$

$$\begin{bmatrix} 5 & 6 \\ 4 & 12 \end{bmatrix} \begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} 54 \\ 72 \end{bmatrix}$$

$$\begin{bmatrix} h \\ s \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

hosta = \$6
 shrub = \$4

② write equations:

$$12A + 8S = 136$$

$$6A + 7S = 83$$

$$\begin{bmatrix} 12 & 8 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} A \\ S \end{bmatrix} = \begin{bmatrix} 136 \\ 83 \end{bmatrix}$$

$$\begin{bmatrix} A \\ S \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

Adult ticket
 cost: \$8
 Student ticket
 cost: \$5

ex Solve the system using matrices:

$$y = 3x - 2$$

$$y = -5x + 14$$

$$y - 3x = -2$$

$$y + 5x = 14$$

$$\begin{bmatrix} 1 & -3 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} -2 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

x = 4 and y = 2