CALCULUS
WORKSHEET ON PARTICLE MOTION

If $s = f(t)$ is the position function of a particle that is moving in a straight line, then the instantaneous velocity of the particle is the rate of change of the displacement with respect to time, and the acceleration of the particle is the rate of change of the velocity with respect to time.

Ex. The position of a particle that is moving in a straight line is given by the equation

$s = t^3 - 6t^2 + 9t$ $\quad v = 3t^2 - 12t + 9 \quad a = 6t - 12$

where t is measured in seconds and s in meters.

(a) Find the velocity at time t.

$v(t) = 3t^2 - 12t + 9$

(b) What is the velocity after 2s? After 4s?

$v(2) = 3(2)^2 - 12(2) + 9 = 12 - 24 + 9 = -3$ m/s

$v(4) = 4(4)^2 - 12(4) + 9 = 64 - 48 + 9 = 25$ m/s

(c) When is the particle at rest?

$v(t) = 0 = 3t^2 - 12t + 9 \quad 0 = 3(t^2 - 4t + 3) = 3(t-3)(t-1) = 0$

$\boxed{t = 3 \quad t = 1}$

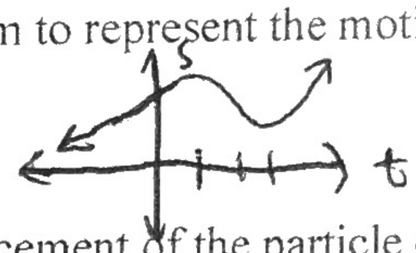
(d) When is the particle moving forward (that is, in the positive direction)?

$v(t) > 0 \quad v(t) = 3(t-3)(t-1)$

		1		3	
	+		−		+
$t-3$	−		−		+
$t-1$	−		+		+

$(-\infty, 1)$
$(3, \infty)$

(e) Draw a diagram to represent the motion of the particle.



(f) Find the displacement of the particle during the first five seconds.

Change from starting position:

$s(0) = 0$

$s(5) = 125 - 150 + 45 = +20$ m

$\Delta s = +20 - 0 = +20$ m

(g) Find the total distance traveled by the particle during the first 5 seconds.

We know that the particle turns around at $t = 1$ and $t = 3$. We need distance from $t = 0$ to $t = 1$, $t = 1$ to $t = 3$, and $t = 3$ to $t = 5$.

$s(0) = 0 \quad s(3) = 0$
$s(1) = 4 \quad s(5) = 20$

Δs from 0 to 1: 4
Δs from 1 to 3: 4
Δs from 3 to 5: 20

$\boxed{\text{total: } 28\text{m}}$

(h) Find the acceleration at time t and after 4s.

$a(t) = 6t - 12$

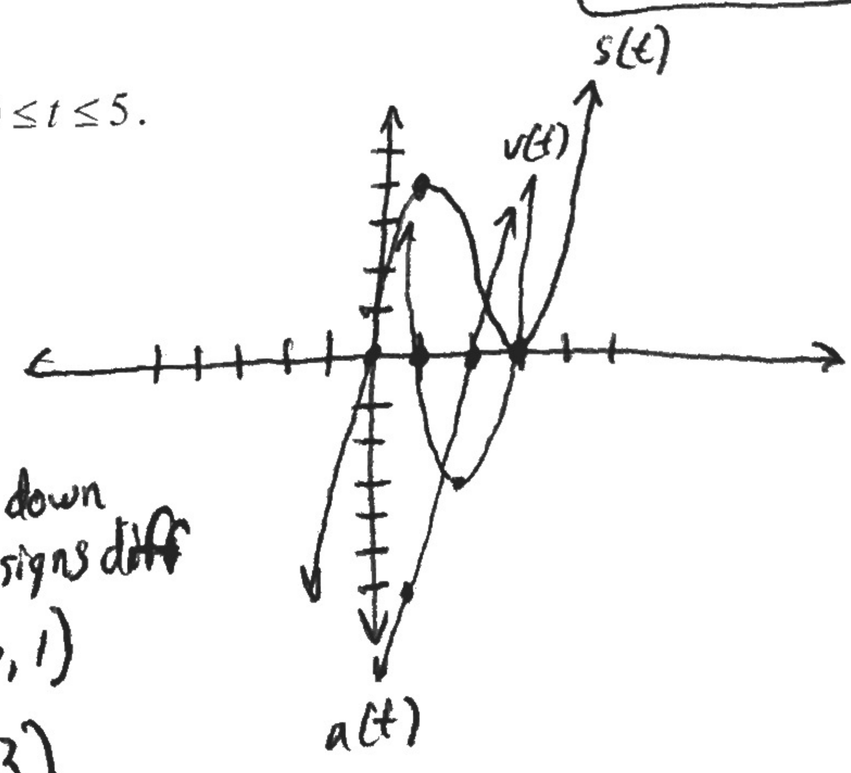
$a(4) = 6(4) - 12 = 12$ m/s^2

(i) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.

$v(t) = 3(t-3)(t-1)$

$a(t) = 6t - 12$

$t = 2$, $a(2) = 0$; and $a(t)$ changes sign. $v(t)$ has min at $t = 2$



(j) When is the particle speeding up? When is it slowing down?

speeding up: $v(t)$ and $a(t)$ have same sign

based on graph { $v(t)$ and $a(t)$ both + $(3, \infty)$; $v(t)$ and $a(t)$ both − $(1, 2)$

slowing down when signs diff $(-\infty, 1)$ $(2, 3)$