$\frac{dy}{dx} \rightarrow$ derivative of y with respect to x.

Implicit Differentiation

We have been able to differentiate functions that are solved for y explicitly up to this point. Now we want to consider functions of the type $x^2 - 2y^3 + 4y = 2$. You can see that it would be quite challenging to solve for y as a function of x explicitly.

Implicit Differentiation

- Realize differentiation is taking place with respect to x.
- When you differentiate terms involving x alone, you can differentiate as usual.
- When you differentiate terms involving y, you must apply the Chain Rule (because you are assuming that y is defined implicitly as a differentiable function of x.)

GUIDELINES FOR IMPLICIT DIFFERENTIATION

1. Differentiate both sides with respect to x.
2. Collect $\frac{dy}{dx}$ terms on one side = to all the other terms.
3. Factor out $\frac{dy}{dx}$.
4. Solve for $\frac{dy}{dx}$.

EX #1: Find $\frac{dy}{dx}$ for $x^2 - y^2 = 16$ at $(-5,3)$

① $2x - 2y\frac{dy}{dx} = 0$

② $-2y\frac{dy}{dx} = -2x$

④ $\frac{dy}{dx} = \frac{-2x}{-2y}$

derivative

$\boxed{\frac{dy}{dx} = \frac{x}{y}}$ @ $(-5,3)$

$\boxed{\frac{-5}{3}}$

Product Rule

F1: $x \rightarrow 1$

F2: $y \rightarrow \frac{dy}{dx}$

EX #2: Find $\frac{dy}{dx}$ for $xy + y = 8$ at $(3,2)$

① $x\left(\frac{dy}{dx}\right) + y(1) + \frac{dy}{dx} = 0$

② $x\frac{dy}{dx} + \frac{dy}{dx} = -y$

③ $\frac{dy}{dx}(x+1) = -y$

④ $\boxed{\frac{dy}{dx} = \frac{-y}{x+1}}$

@ $(3,2)$

$\hookrightarrow \frac{-2}{3+1} = \boxed{-\frac{1}{2}}$

Product Rule!

F1: $3x \rightarrow 3$

F2: $y \rightarrow dy/dx$ derivative

EX #3: Find the instantaneous rate of change at $(1,1)$ for $x + 3xy - 2y^2 = 2$

① $1 + (3x)\left(\frac{dy}{dx}\right) + y(3) - 4y\frac{dy}{dx} = 0$

② $3x\frac{dy}{dx} - 4y\frac{dy}{dx} = -1 - 3y$

③ $\frac{dy}{dx}(3x - 4y) = -1 - 3y$

④ $\boxed{\frac{dy}{dx} = \frac{-1-3y}{3x-4y}}$

$\hookrightarrow$ @ $(1,1) \rightarrow \frac{-4}{-1} = \boxed{4}$

EX #4: Find $\frac{dy}{dx}$ given that $y^3 + 5y^2 - 5y - x^2 = -4$

① $3y^2\frac{dy}{dx} + 10y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x = 0$

② $3y^2\frac{dy}{dx} + 10y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$

③ $\frac{dy}{dx}(3y^2 + 10y - 5) = 2x$

④ $\boxed{\frac{dy}{dx} = \frac{2x}{3y^2 + 10y - 5}}$

EX #5: Find $\frac{dy}{dx}$ for $x + \sqrt{xy} = 6$ at $(3,3)$

Chain Rule! & product Rule

out: $(\text{something})^{1/2}$

inside: $xy \rightarrow \left(x\frac{dy}{dx} + y(1)\right)$

① $1 + \frac{1}{2}(xy)^{-1/2} \cdot \left(x\frac{dy}{dx} + y\right) = 0$

② $\left(\frac{1 \cdot x}{2(xy)^{1/2}}\right)\left(\frac{dy}{dx}\right) + \frac{1 \cdot y}{2(xy)^{1/2}} = -1$

$\frac{x}{2(xy)^{1/2}}\left(\frac{dy}{dx}\right) = -1 - \frac{y}{2(xy)^{1/2}}$

$\frac{dy}{dx} = -\frac{2\sqrt{xy}}{x} - \frac{y \cdot 2\sqrt{xy}}{x\, 2\sqrt{xy}}$

$\boxed{\frac{dy}{dx} = \frac{-2\sqrt{xy}}{x} - \frac{y}{x}}$

@ $(3,3)$

$\hookrightarrow \frac{-2(3)}{3} - \frac{3}{3}$

$-2 - 1 = \boxed{-3}$

EX #6: Find $\frac{dy}{dx}$ for $(x-y)^2 + y = 6$ at $(0,2)$

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