Day 1 – Optimization Notes

THINK ABOUT IT - An open rectangular box is formed by cutting congruent squares from the corners of a piece of cardboard and folding the sides up. The original piece of cardboard was 20 x 18 cm

a. What is the relative domain?

$\frac{18}{2} = 9$ $(0, 9)$ $0 < x < 9$

b. Write an expression to model the volume in terms of x?

$(20-2x)(18-2x)x = V$

c. What value of x will yield the maximum volume?

$x = 3.15$ $y = 504.91$

d. What is the maximum volume?

$504.91\ cm^3$

e. What is the relative range?

$(0, 504.91)$

f. What size square must be removed to yield a volume of 242 cm^3

$242 = (20-2x)(18-2x)x$

Graph $y = 242$
Find intersect

$\boxed{x = 6.38 \quad .802}$

Optimization - ______________________________

Steps to solving an optimization problem:

- Understand the problem. Read the problem carefully. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?
- Draw a Diagram.
- Introduce variables.
- Write a primary equation for the quantity to be optimized
- Write a secondary equation to help eliminate one of the variables
- Reduce the primary equation to one having a single independent variable.
- Determine the feasible domain.
- Solve

Examples:

1. A rectangular garden is to be enclosed using the wall of a building as one side and 60-feet of fencing on the other three sides. Find the length and width that will give the maximum area.

x x y

$2x + y = 60$ $A = xy$ $\boxed{x = 15}$, $A = 450$

$y = 60 - 2x$ $A = x(60-2x)$ $\boxed{y = 30}$

2. A carpenter is building a rectangular room with a perimeter of 68 feet. What dimensions will yield the maximum floor area? What is the maximum floor area?

$2L + 2W = 68$

$A = LW$

$2L = 68 - 2W$

$\boxed{L = 17ft}$ $L = 34 - W$

$A = (34-W)(W)$

$\boxed{W = 17ft}$, $\boxed{A = 289}$ ft^2

ICM – Unit 3 – Optimization, Logarithms, & Exponentials

3. The sum of two number is 6. The sum of their squares is a minimum. What are the two numbers?

$x + y = 6$ $y = 6 - x$

$x^2 + y^2 = S$

$x^2 + (6-x)^2 = S$ $\rightarrow \boxed{x = 3, \text{ so } y = 3}$

4. The sum of a positive number and 4 times the square of its reciprocal is a minimum. What is the number?

$x + 4x^{-2} = S$

$x = 2$

5. An open rectangular box has square base and a volume of 400 cubic inches. What dimensions minimize the amount of cardboard need to make the box?

WINDOW
-100
100
-100
500

$V = 400\ in^3$ $SA = x^2 + 4yx$

$400 = x^2 y$ $SA = x^2 + 4\left(\frac{400}{x^2}\right)x$

$\frac{400}{x^2} = y$ $SA = x^2 + \frac{1600}{x}$

$9.28 \times 9.28 \times 4.64$ $x = 9.28$ $y = 4.64$

Homework

1. The product of two positive numbers is 6. Their sum is minimum. What are the two numbers?

$xy = 6$ $y = \frac{6}{x}$ $x + \frac{6}{x} = S$

$x + y = S$

$x = 2.45, y = 2.45$

2. A rectangular piece of land boarders a wall. The land is to be enclosed and divided by 200 feet of fencing as shown below. (No fence is along the wall.) What is the largest area that can be enclosed?

$4x + y = 200$ $y = 200 - 4x$

$xy = A$ $x(200 - 4x) = A$

x x x x $A = 25$

$x = 25$, $y = 100$

3. An open rectangular box has a square base and a volume of 500 cubic inches. What dimension minimize the amount of cardboard.

WINDOW

$V = 500\ in^3$ $x = 9.99$

$500 = x^2 y$ $SA = x^2 + 4xy$ $y = 5$

$\frac{500}{x^2} = y$ $= x^2 + 4x\left(\frac{500}{x^2}\right)$ $10 \times 10 \times 5$

4. An open rectangular box is formed by cutting congruent squares from the corners of a piece of cardboard and folding the sides up. If the original piece of cardboard was 24 by 45 inches, what are the dimensions of the bow with maximum volume?

$x(24-2x)(45-2x) = V$

$x = 12, 22.5$

24 45

max $\rightarrow$ $x = 5$

L W H
$14 \times 35 \times 5$

$24 - 2(5) = 14$ $45 - 2(5) = 35$