Average Rate of Change

Average Rate of Change

- Functions are often used to model changing quantities.

- In this section, we learn how to:

  - Find the rate at which the values of a function change as the input variable changes.

- Average Rate of Change

Average Rate of Change

- We are all familiar with the concept of speed.
  - If you drive a distance of 120 miles in 2 hours, then your average speed, or rate of travel,
    is:

$$\frac{120 \text{ mi}}{2 \text{ h}} = 60 \text{ mi/h}$$

Average Rate of Change

- Now, suppose you take a car trip and record the distance that you travel every few minutes.

  - The distance s you have traveled is a function of the time t:

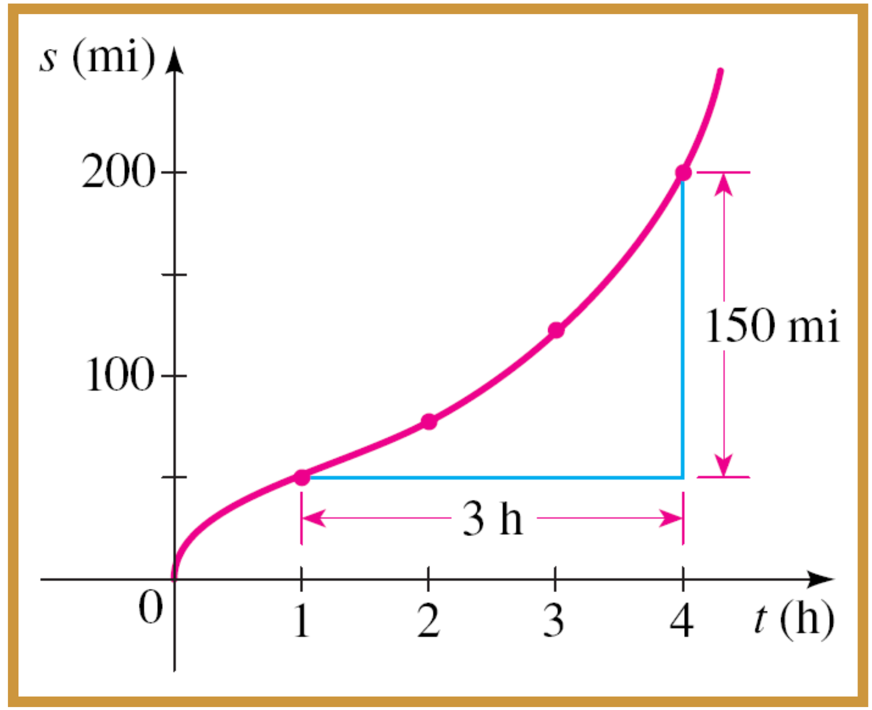
$$s(t) = \text{total distance traveled at time } t$$

Average Rate of Change

•We graph the function *s* as shown.

- The graph shows that you have traveled a total of:

  50 miles after 1 hour
  75 miles after 2 hours
  140 miles after 3 hours
  and so on.



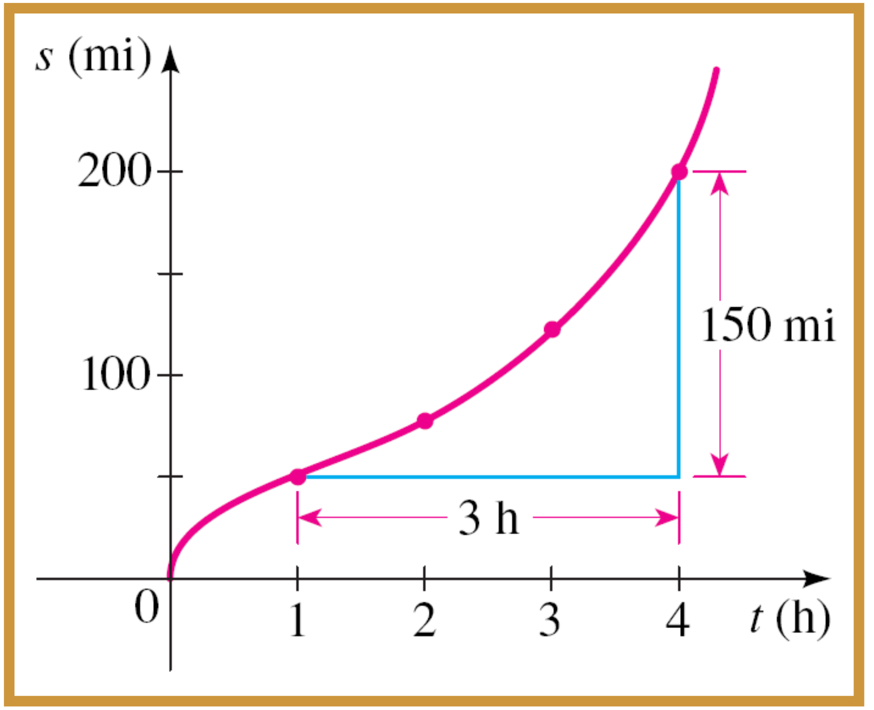
Average Rate of Change

- To find your average speed between any two points on the trip, we divide the distance traveled by the time elapsed.
    - Let’s calculate your average speed between 1:00 P.M. and 4:00 P.M.
    - The time elapsed is 4 – 1 = 3 hours.

Average Rate of Change

- To find the distance you traveled, we subtract the distance at 1:00 P.M. from the distance at 4:00 P.M.,
  that is,

  200 – 50 = 150 mi



Average Rate of Change

- Thus, your average speed is:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

$$= \frac{150 \text{ mi}}{3 \text{ h}}$$

$$= 50 \text{ mi/h}$$

Average Rate of Change

- The average speed we have calculated can be expressed using function notation:

$$\text{average speed} = \frac{s(4) - s(1)}{4 - 1}$$

$$= \frac{200 - 50}{3}$$

$$= 50 \text{ mi/h}$$

Average Rate of Change

- Note that the average speed is different over different time intervals.

Average Rate of Change

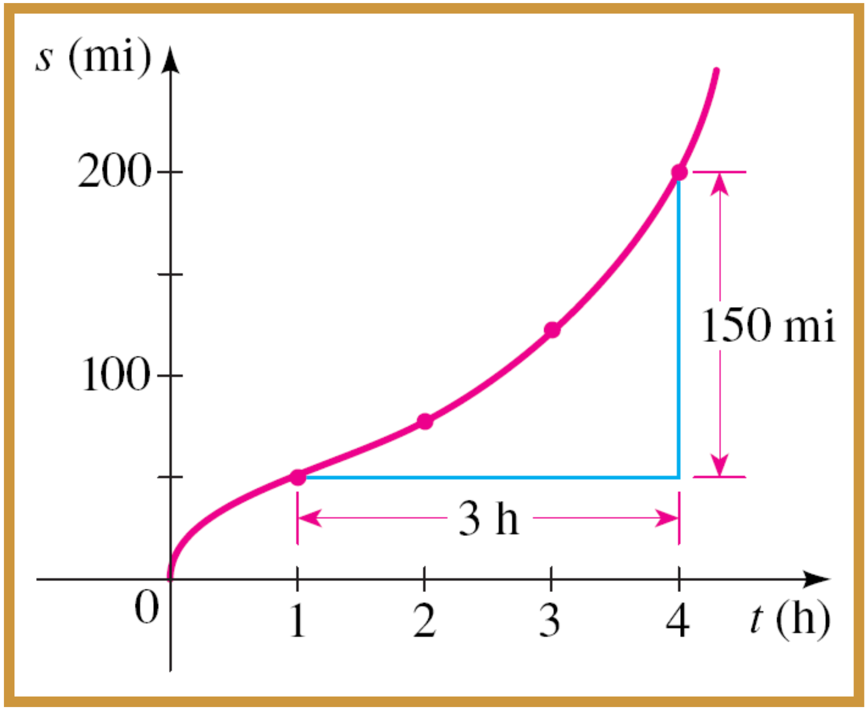
- For example, between 2:00 P.M. and 3:00 P.M., we find that:

average speed

$$= \frac{s(3) - s(2)}{3 - 2}$$

$$= \frac{140 - 75}{1}$$

$$= 65 \text{ mi/h}$$



Average Rate of Change—Significance

- Finding average rates of change is important in many contexts.
- For instance, we may be interested in knowing:
  - How quickly the air temperature is dropping as a storm approaches.
  - How fast revenues are increasing from the sale of a new product.

Average Rate of Change—Significance

- So, we need to know how to determine the average rate of change of the functions that model these quantities.

  - In fact, the concept of average rate of change can be defined for any function.

Average Rate of Change—Definition

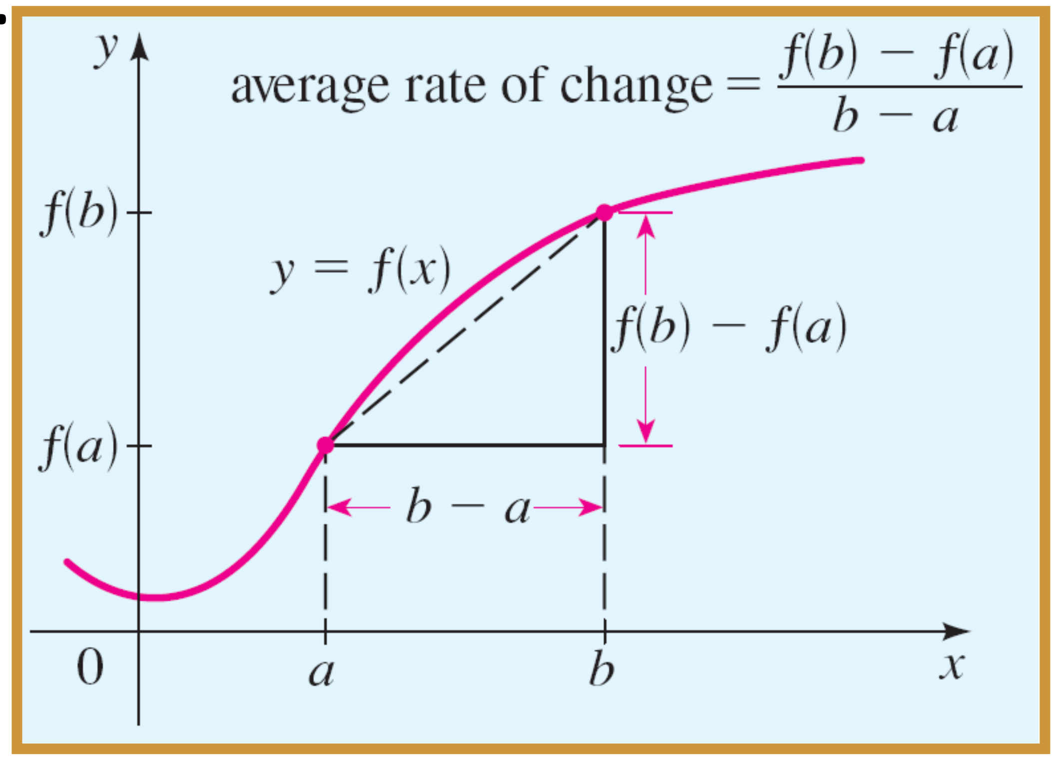
- The average rate of change of the function $y = f(x)$ between $x = a$ and $x = b$ is:

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{f(b) - f(a)}{b - a}$$

Average Rate of Change—Definition

- The average rate of change is the slope of the secant line between *x* = *a* and *x* = *b* on the graph of *f*.
  - This is the line that passes through (*a*, *f*(*a*)) and (*b*, *f*(*b*)).

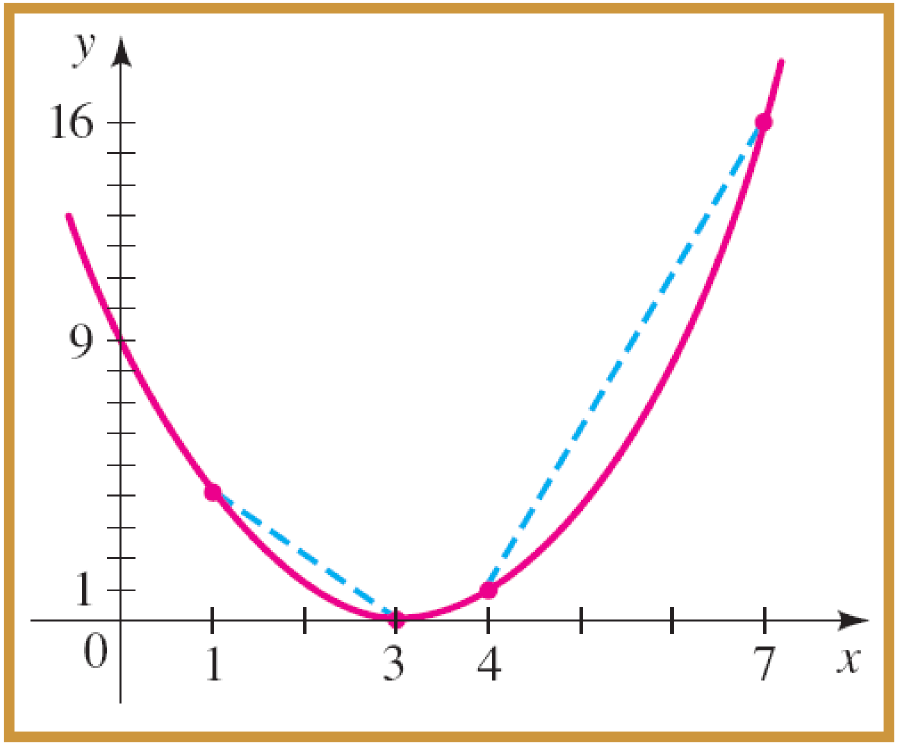


E.g. 1—Calculating the Average Rate of Change

- For the function $f(x) = (x - 3)^2$, whose graph is shown, find the average rate of change between the following points:

(a) $x = 1$ and $x = 3$

(b) $x = 4$ and $x = 7$



E.g. 1—Average Rate of Change

Example (a)

Average rate of change

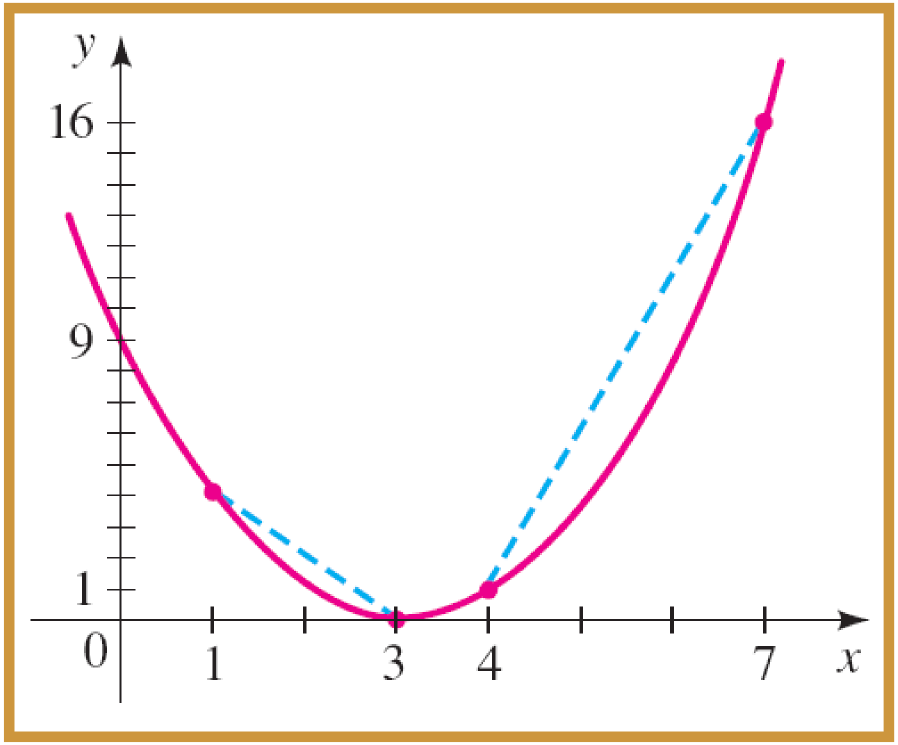
$$= \frac{f(3) - f(1)}{3 - 1}$$

$$= \frac{(3-3)^2 - (1-3)^2}{3-1}$$

Use $f(x) = (x-3)^2$

$$= \frac{0-4}{2}$$

$$= -2$$



Example (b)

Average rate of change

$$= \frac{f(7) - f(4)}{7 - 4}$$

$$= \frac{(7-3)^2 - (4-3)^2}{7-4}$$

$\left(\text{Use } f(x) = (x-3)^2\right.$

$$= \frac{16 - 1}{3}$$

$$= 5$$