Exercise 1.1

Differentiate the following functions using the composite function rule.

a. $(2x + 3)^2$ b. $(x^2 + 2x + 1)^{12}$ c. $(3 - x)^{21}$

d. $(x^3 - 1)^5$ e. $f(t) = \sqrt{t^2 - 5t + 7}$ f. $g(z) = \frac{1}{\sqrt{2 - z^4}}$

g. $y = (t^3 - \sqrt{t})^{-3.8}$ h. $z = (x + \frac{1}{x})^{\frac{3}{7}}$

Exercise 1.2

Differentiate the functions below. You will need to use both the composite function rule and the product or quotient rule.

a. $(x + 2)(x + 3)^2$ b. $(2x - 1)^2(x + 3)^3$ c. $x\sqrt{(1 - x)}$

d. $x^{\frac{1}{3}}(1 - x)^{\frac{2}{3}}$ e. $\frac{x}{\sqrt{1 - x^2}}$

Solutions to exercises

Exercise 1.1

a. $\dfrac{d}{dx}\left((2x+3)^2\right) = 8x + 12$

b. $\dfrac{d}{dx}\left((x^2+2x+1)^{12}\right) = 12(x^2+2x+1)^{11}(2x+2)$

c. $\dfrac{d}{dx}\left((3-x)^{21}\right) = -21(3-x)^{20}$

d. $\dfrac{d}{dx}\left((x^3-1)^5\right) = 5(x^3-1)^4 3x^2 = 15x^2(x^3-1)^4$

e. $\dfrac{d}{dt}\sqrt{t^2-5t+7} = \dfrac{d}{dt}(t^2-5t+7)^{\frac{1}{2}} = \dfrac{1}{2}(t^2-5t+7)^{-\frac{1}{2}}(2t-5)$

f. $\dfrac{d}{dz}\left(\dfrac{1}{\sqrt{2-z^4}}\right) = \dfrac{d}{dz}\left((2-z^4)^{-\frac{1}{2}}\right) = 2z^3(2-z^4)^{-\frac{3}{2}}$

g. $\dfrac{d}{dt}\left((t^3-\sqrt{t})^{-3.8}\right) = -3.8(t^3-\sqrt{t})^{-4.8}\left(3t^2 - \dfrac{1}{2\sqrt{t}}\right)$

h. $\dfrac{d}{dx}\left((x+\tfrac{1}{x})^{\frac{3}{7}}\right) = \dfrac{3}{7}(x+\tfrac{1}{x})^{-\frac{4}{7}}(1-\dfrac{1}{x^2})$

Exercise 1.2

a. $\dfrac{d}{dx}\left((x+2)(x+3)^2\right) = (x+3)^2 + 2(x+2)(x+3)$

b. $\dfrac{d}{dx}\left((2x-1)^2(x+3)^3\right) = 4(2x-1)(x+3)^3 + 3(2x-1)^2(x+3)^2$

c. $\dfrac{d}{dx}\left(x\sqrt{1-x}\right) = \sqrt{1-x} - \dfrac{x}{2\sqrt{1-x}}$

d. $\dfrac{d}{dx}\left(x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}\right) = \dfrac{1}{3}x^{-\frac{2}{3}}(1-x)^{\frac{2}{3}} - \dfrac{2}{3}x^{\frac{1}{3}}(1-x)^{-\frac{1}{3}}$

e. $\dfrac{d}{dx}\left(\dfrac{x}{\sqrt{1-x^2}}\right) = \dfrac{\sqrt{1-x^2}+x^2(1-x^2)^{-\frac{1}{2}}}{1-x^2}$