

ex: $x + \overbrace{3xy}^{\text{product rule}} - 2y^2 = 2$ at (1,1)

$$1 + 3x \cdot 1 \frac{dy}{dx} + y^3 - 4y \frac{dy}{dx} = 0$$

$$1 + 3x \frac{dy}{dx} + 3y - 4y \frac{dy}{dx} = 0$$

$$3x \frac{dy}{dx} - 4y \frac{dy}{dx} = -1 - 3y$$

$$\frac{dy}{dx} \frac{(3x - 4y)}{(3x - 4y)} = \frac{-1 - 3y}{(3x - 4y)}$$

$$\frac{dy}{dx} = \frac{-1 - 3y}{(3x - 4y)} \quad \frac{-1 - 3(1)}{(3(1) - 4(1))} = \frac{-4}{-1} = \textcircled{4}$$

ex: $\overbrace{3x^2y}^{\text{product rule}} + \overbrace{2x}^{\text{product rule}} - \overbrace{y^2}^{\text{product rule}} = \overbrace{7}^{\text{product rule}}$

$$3x^2 \left(1 \frac{dy}{dx} \right) + y(6x) + 2 - 2y \frac{dy}{dx} = 0$$

$$3x^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = -2 - 6xy$$

$$\frac{dy}{dx} \frac{(3x^2 - 2y)}{(3x^2 - 2y)} = \frac{-2 - 6xy}{(3x^2 - 2y)}$$

$$\frac{dy}{dx} = \frac{-2 - 6xy}{(3x^2 - 2y)} \quad \checkmark$$

ex: Find $\frac{d^2y}{dx^2}$ of $x^2 + y^2 = 25$

First derivative: $2x + 2y \frac{dy}{dx} = 0$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

Second derivative: Start over!

$$\frac{dy}{dx} = \frac{-x}{y} \text{ high quotient rule!}$$

low

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)(1 \frac{dy}{dx})}{(y)^2}$$

Problem!!! There's still a $\frac{dy}{dx}$ in the answer.....

BUT earlier we said $\frac{dy}{dx} = \frac{-x}{y}$ so sub it in!

$$\frac{d^2y}{dx^2} = \frac{-y + x \left(\frac{-x}{y} \right)}{y^2} = \frac{-y - \frac{x^2}{y}}{y^2}$$