① $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 8 & \text{if } x = 3 \end{cases}$

$\underline{x = 3}$

① $f(3) = 8$

② $\lim_{x \to 3} f(x) \Rightarrow 6$

$\dfrac{x^2-9}{x-3} = \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = x+3 \rightarrow 3+3=6$

Discontinuous @ $x = 3$

③ $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$

-1

① $f(-1) = -1$

② $\lim_{x \to -1} f(x) = -1$

$\lim_{x \to -1^-} f(x) = -1$ $\quad$ $\lim_{x \to -1^+} f(x) = -1$

③ $f(-1) = \lim_{x \to -1} f(x)$

$-1 = -1$ ✓

Continuous @ $x = -1$

1

① $f(1) = 0$

② $\lim_{x \to 1} = DNE$

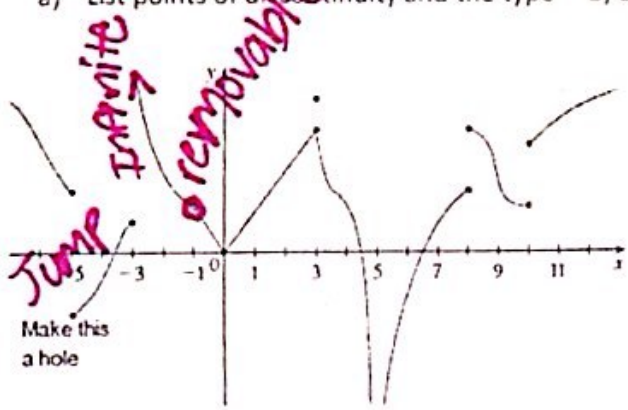
$\lim_{x \to 1^-} f(x) = 1$ $\quad$ $\lim_{x \to 1^+} f(x) = 0$

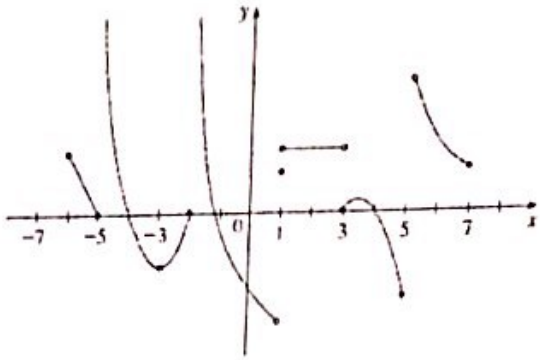
Discontinuous @ $x = 1$

Continuity

For the graphs below,

a) List points of discontinuity and the type b) List the intervals for which the function is continuous





State the Three Conditions for Continuity:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $f(c) = \lim_{x \to c} f(x)$

In the following, use the definition to determine if the function is continuous. If discontinuous tell which part of the definition it failed. (Hint, you only need to check end points)

1 $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 8 & \text{if } x = 3 \end{cases}$	$f(x) = \begin{cases} 2x-3 & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$ Discontinuous at $x=2$	3 $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ 1-x & \text{if } x \geq 1 \end{cases}$
$g(x) = \begin{cases} x^2+4, & x \leq -1 \\ 4+x, & x > -1 \end{cases}$ Discontinuous @ $x=-1$	$f(x) = \begin{cases} 2x+1 & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x < 1 \\ 2x-1 & \text{if } x \geq 1 \end{cases}$ Discontinuous @ $x=-1$ & $x=1$	$f(x) = \begin{cases} (x-1)^3 & \text{if } x < 0 \\ (x+1)^3 & \text{if } x \geq 0 \end{cases}$ Discontinuous @ $x=0$
$f(x) = \begin{cases} 1/x & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1/x^2 & \text{if } x > 1 \end{cases}$ continuous	$f(x) = \begin{cases} \sqrt{x+1}, & 0 \leq x \leq 3 \\ 5-x, & 3 < x \leq 5 \end{cases}, \quad x=3.$ continuous	$f(x) = \begin{cases} -2x+3, & x < 1 \\ x^2, & x \geq 1 \end{cases}, \quad x=1.$ continuous