

# Synthetic Division

- ① Divide  $3x^3 - 2x^2 + 3x - 4$  by  $x-3$  using synthetic division.

$$\begin{aligned}x-3 &= 0 \\ x &= 3\end{aligned}$$

$$\begin{array}{r|rrrr} 3 & 3 & -2 & 3 & -4 \\ & \downarrow & \nearrow 9 & \nearrow 21 & \nearrow 68 \\ \hline & 3 & 7 & 24 & \boxed{68} \end{array}$$

$$3x^2 + 7x + 24 + \frac{68}{x-3}$$

- ② Use synthetic division to determine whether  $x-4$  is a factor of  $-2x^5 + 6x^4 + 10x^3 - 6x^2 - 9x + 4$

$$\begin{array}{r|rrrrrr} 4 & -2 & 6 & 10 & -6 & -9 & 4 \\ & \downarrow & -8 & -8 & 8 & 8 & -4 \\ \hline & -2 & -2 & 2 & 2 & -1 & \boxed{0} \end{array}$$

YES,  $x-4$  is a factor of  $-2x^5 + 6x^4 + 10x^3 - 6x^2 - 9x + 4$  because the remainder is zero.

③ Test to see if 4 is a root of  $x^3 - 12x^2 + 48x - 64$ .

$$4 \begin{array}{r|rrrr} 1 & -12 & 48 & -64 \\ \downarrow & 4 & -32 & 64 \\ \hline 1 & -8 & 16 & 0 \end{array}$$

$$4 \begin{array}{r|rr} 1 & -8 & 16 \\ \downarrow & 4 & -16 \\ \hline 1 & -4 & 0 \end{array}$$

$$4 \begin{array}{r|r} 1 & -4 \\ \downarrow & 4 \\ \hline 1 & 0 \end{array}$$

$$1 \quad 0$$

Yes!  
4 is a root.

This tells us that  
→ 4 is a double root

→ 4 is a TRIPLE ROOT

# Rational Root Theorem

★ RRT is a way to list possible rational roots. Divide the factors of the constant term by the factors of the leading coefficient to get possible roots. Test with synthetic division.

① Find the roots of  $x^3 - 2x^2 - x + 2$  → Possible rational roots

Factors of 2 : 2, 1 =  $\pm 1, \pm 2$   
Factors of 1 : 1  
↳ Don't forget  $\pm$

2		1	-2	-1	2
		↓	2	0	-2
		1	0	-1	0

2 is a root

$$x^2 - 1 = 0$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{\pm \sqrt{4}}{2}$$

$$x = \frac{\pm 2}{2}$$

$$x = \pm 1$$

Quadratic Formula will give us our 2 other roots whether they're rational or not.

$$x = 2, -1, 1$$

② Find all roots :  $3x^3 - 11x^2 + 5x + 3$

$$\begin{array}{l} \text{Factors of } 3: \quad 1, 3 \\ \hline \text{Factors of } 3: \quad 1, 3 \end{array} = \pm 1, \pm \frac{3}{1}, \pm \frac{1}{3}, \pm \frac{3}{3}$$

or  $\pm 1, \pm 3, \pm \frac{1}{3}$

$$x = 3, 1, -\frac{1}{3}$$

# Applications of Polynomials

- ① The difference between 2 integers is 7 and their product is 120. Find the integers.

$$x - y = 7 \longrightarrow x = 7 + y$$

$$xy = 120$$

$$(7 + y)(y) = 120$$

$$7y + y^2 = 120$$

$$y^2 + 7y - 120 = 0$$

$$y = \frac{-7 \pm \sqrt{7^2 - 4(1)(-120)}}{2(1)}$$

$$y = \frac{-7 \pm \sqrt{49 + 480}}{2}$$

$$y = \frac{-7 \pm \sqrt{529}}{2} \rightarrow y = \frac{-7 \pm 23}{2}$$

$$y = \frac{-7 + 23}{2}$$

$$y = 8$$

$$y = \frac{-7 - 23}{2}$$

$$y = -15$$

$$\begin{aligned} x - 8 &= 7 \\ +8 &+8 \\ \hline x &= 15 \end{aligned}$$

$$\begin{aligned} x - (-15) &= 7 \\ x + 15 &= 7 \\ \hline x &= -8 \end{aligned}$$

$$\boxed{x = 15, y = 8}$$

$$\boxed{x = -8, y = -15}$$

BOTH WORK