Date ________
Notes: 7.8

Warm up:

1. Solve this system of equations using elimination: $\begin{cases} 4x - y = 10 \\ 5x + 2y = 6 \end{cases}$

$2 \quad 8x - 2y = 20$
$+ \; 5x + 2y = 6$
$\frac{13x}{13} = \frac{26}{13}$
$x = \frac{26}{13}$

$\left(\frac{26}{15}, \frac{-46}{15}\right)$

2a. If you were to solve a system of equations by graphing, where is the solution located on the graph?

Point

2b. How does a graph look when there is no solution to the system of equations?

Parallel

2c. How does the graph look when there is an infinite number of solutions to the system of equations?

Same line

Essential Questions:
1. How do geometric relationships help us to solve problems and make sense of our world?
2. How do we use math models to describe relationships?

Learning Targets:
7.8.1 I can solve a system of equations using a matrix.
7.8.2 I recognize when a system of equations has 0, 1, or infinitely many solutions using a matrix.
7.8.3 I can find the solution to an application problem (2 or more variables) using a matrix.

ICM

Name________________________ ID: 1

Matrices- Determinant, Inverse

Date________________ Period____

Evaluate each determinant.

1) $\begin{vmatrix} 1 & -5 \\ -2 & -5 \end{vmatrix}$ $-5-10=\boxed{-15}$

2) $\begin{vmatrix} -4 & 4 \\ 3 & -4 \end{vmatrix}$ $16-12=\boxed{4}$

3) $\begin{vmatrix} -1 & -1 \\ -5 & 4 \end{vmatrix}$ $-4-5=\boxed{-9}$

4) $\begin{vmatrix} -2 & -1 \\ -3 & -5 \end{vmatrix}$ $10-3=\boxed{7}$

5) $\begin{vmatrix} -3 & 5 & 2 \\ 3 & -1 & -4 \\ -2 & -5 & -1 \end{vmatrix}$ $\begin{matrix} -3 & 5 \\ 3 & -1 \\ -2 & -5 \end{matrix}$ 78

$(-3)(40)(-30)-(4)(-60)(-15)$

6) $\begin{vmatrix} -1 & 1 & 3 \\ 1 & -5 & 5 \\ 5 & 3 & -4 \end{vmatrix}$ $\boxed{108}$

7) $\begin{vmatrix} 2 & 0 & -1 \\ 2 & 4 & 0 \\ 0 & 4 & -5 \end{vmatrix}$ -48

8) $\begin{vmatrix} 2 & 4 & 0 \\ 4 & 4 & -4 \\ 2 & 5 & 5 \end{vmatrix}$ $\boxed{-32}$

For each matrix state if an inverse exists.

9) $\begin{bmatrix} 6 & 0 \\ -9 & 0 \end{bmatrix}$ $0-0=0$ No!

10) $\begin{bmatrix} 4 & -9 \\ -1 & 3 \end{bmatrix}$ $12-9=3$ yes!

Find the inverse of each matrix.

$\frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

11) $\begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix}$ $-6-4=-10$

$-\frac{1}{10}\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} -2/10 & -1/10 \\ -2/5 & 3/10 \end{bmatrix}$

12) $\begin{bmatrix} 2 & 6 \\ -2 & -5 \end{bmatrix}$ $\begin{bmatrix} -5/2 & -3 \\ 1 & 1 \end{bmatrix}$

13) $\begin{bmatrix} -1 & 2 \\ 5 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1/5 \\ 1/2 & 1/10 \end{bmatrix}$

14) $\begin{bmatrix} -6 & 4 \\ -1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & -1 \\ 1/4 & -3/2 \end{bmatrix}$

Solve each equation or state if there is no unique solution.

15) $\begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix} Z = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$ $\boxed{\begin{bmatrix} 7 \\ -10 \end{bmatrix}}$

$Z=\begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix}^{-1}\begin{bmatrix} -10 \\ 5 \end{bmatrix}$

16) $\begin{bmatrix} 6 & -3 \\ 9 & -4 \end{bmatrix} Y = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$ $\begin{bmatrix} -3 \\ -8 \end{bmatrix}$

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17) $\begin{bmatrix} 9 & -4 \\ 0 & 0 \end{bmatrix} X = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$

None!

18) $\begin{bmatrix} -19 \\ -6 \end{bmatrix} = \begin{bmatrix} -3 \\ -8 \end{bmatrix} + \begin{bmatrix} 7 & -4 \\ -2 & 2 \end{bmatrix} C$

$\begin{bmatrix} -16 \\ 2 \end{bmatrix}$ $\boxed{\begin{bmatrix} -4 \\ -3 \end{bmatrix}}$

19) $\begin{bmatrix} -6 & 1 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix} X = \begin{bmatrix} -3 & 42 \\ -15 & 1 \end{bmatrix}$

$\begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix} X = \begin{bmatrix} 3 & 41 \\ -15 & +8 \end{bmatrix}$

$\begin{bmatrix} -1 & 10 \\ -8 & 9 \end{bmatrix}$

20) $\begin{bmatrix} -1 & 11 \\ 8 & -6 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 1 & 5 \end{bmatrix} X = \begin{bmatrix} -13 & -1 \\ 40 & 10 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 \\ 1 & 5 \end{bmatrix} X = \begin{bmatrix} -12 & -12 \\ 32 & 16 \end{bmatrix}$

$\boxed{\begin{bmatrix} 7 & 11 \\ 5 & 1 \end{bmatrix}}$

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Solving Systems of Equations in Three Variables:

Example 1 Solve this system of equations using matrices:

$$\begin{cases} 3x + y - z = -6 \\ 2x - y + 2z = 8 \\ 4x + y - 3z = -21 \end{cases}$$

Step 1: Enter the augmented matrix.

$$\begin{bmatrix} 3 & 1 & -1 & -6 \\ 2 & -1 & 2 & 8 \\ 4 & 1 & -3 & -21 \end{bmatrix}$$

$$\boxed{\begin{array}{l} x = -1 \\ y = 4 \\ z = 7 \end{array}}$$

Step 2: Solve the system using "rref" (reduced row echelon form).

Step 3: Interpret your findings:

You Try: Solve the systems below:

1. $\begin{cases} 2x - y + 2z = 15 \\ y + z - 3 = x \\ 3x - y - 18 = -2z \end{cases}$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Ans: 3 1 5

Interpretation:

$x = 3$

$y = 1$

$z = 5$

2. $\begin{cases} 4x + 4y - 2z = 8 \\ 3x - 5y + 3z = 0 \\ 2x + 2y - z = 4 \end{cases}$

$x + \frac{1}{16} z = 5/4$

$y - 9/16 z = 3/4$

$$\begin{bmatrix} 1 & 0 & 1/16 & 5/4 \\ 0 & 1 & -9/16 & 3/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans:

Interpretation:

Inf. many

3. $\begin{cases} 2x + 3y - 8z = 10 \\ z - 4y = 1 \\ -2x - 3y + 8z = 5 \end{cases}$

Ans: No intersection

Interpretation:

$$\begin{bmatrix} 1 & 0 & -29/8 & 0 \\ 0 & 1 & -1/4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7.8.3 I can find the solution to an application problem (2 or more variables) using a matrix.

The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases for \$99 on Monday. On Tuesday they sold 4 balls, 8 bats, and 2 bases for \$78. On Wednesday they sold 2 balls, 3 bats, and 1 base for \$33.60. What are the prices of 1 ball, 1 bat, and 1 base?

x y z

First define the variables.

Balls = X
Bats = y
Bases = Z

Translate the information in the problem into three equations.

$10x + 3y + 2z = 99$
$4x + 8y + 2z = 78$
$2x + 3y + 1z = 3360$

Set up your augmented matrix and interpret the results in context.

Balls = \$8
Bats = \$5.40
Bases = \$1.40

You try:

At the arcade, Ryan, Sara and Tim played video racing games, pinball, and air hockey. Ryan spent \$6 for 6 racing games, 2 pinball games, and 1 game of air hockey. Sara spent \$12 for 3 racing games, 4 pinball games, and 5 games of air hockey. Tim spent \$12.25 for 2 racing games, 7 pinball games, and 4 games of air hockey. How much did each of the games cost?

Use the process outlined above.

$6R + 2P + A = 6$
$3R + 4P + 5A = 12$
$2R + 7P + 4A = 12.25$

R = 0.50
P = 0.75
A = \$1.50