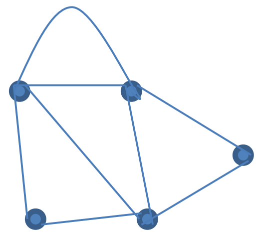
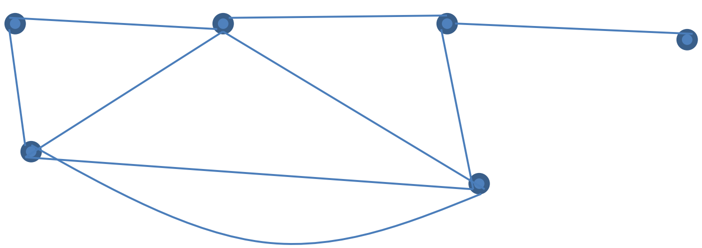
Warm -Up

1. Draw a connected graph with 4 vertices and 7 edges. What is the sum of the degrees of all the vertices?

1. Is this graph a. traceable? b. Eulerian?



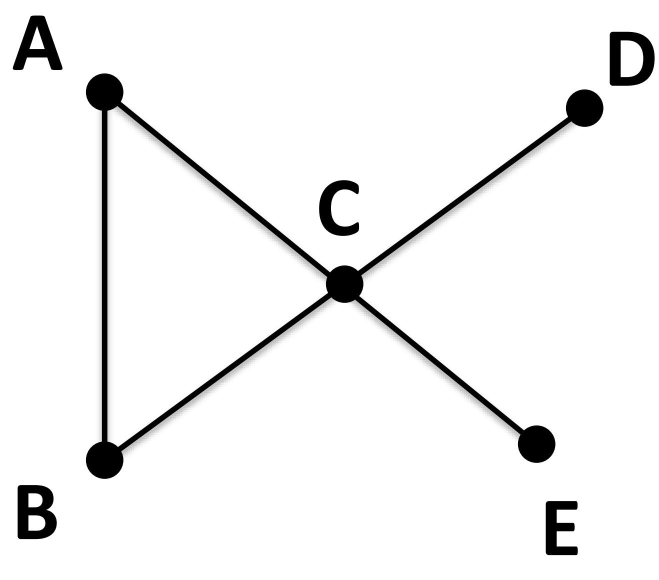
3. Eulerize this graph.



Warm-Up

Eulerize this graph.

Remember: you cannot create NEW edges, only duplicate old ones.



Graph Theory:
Hamilton Circuits and the Traveling Salesperson

☆ **Intro: Traveling Salesperson Example (TSP)**

Danielle is a regional sales manager for a publishing company, lives in Philadelphia, and must make visits next week to branch offices in New York City, Cleveland, Atlanta, and Memphis. In order to find the cheapest trip, she has found the prices of flights between each pair of cities. How should she travel between the cities to keep her costs at the lowest value?

☆ Model Danielle's trip with a graph:

Flight Prices

NYC > Cleveland	**$350**
NYC > Memphis	**$310**
NYC > Philadelphia	**$210**
NYC > Atlanta	**$250**
Cleveland > Philadelphia	**$420**
Cleveland > Atlanta	**$290**
Cleveland > Memphis	**$240**
Memphis > Philadelphia	**$280**
Memphis > Atlanta	**$170**
Atlanta > Philadelphia	**$230**

☆ Find 2 paths that Danielle could take if she leaves from Philadelphia and must return there by the end of the week.

☆ A __path______that crosses through all of the ______vertices________ exactly __once_____ is called a **Hamilton path**.

☆ If a _____Hamilton______ ___path_____ begins and _____ends______ at the same __vertex__ it is called a **Hamilton Circuit**.

☆ Hmm…the definition of a Hamilton path sounds very similar to the definition of an Euler path. What *is* the difference between an Euler path and a Hamilton path???

Euler path: ALL EDGES ONCE

Hamilton path: ALL VERTICES ONCE

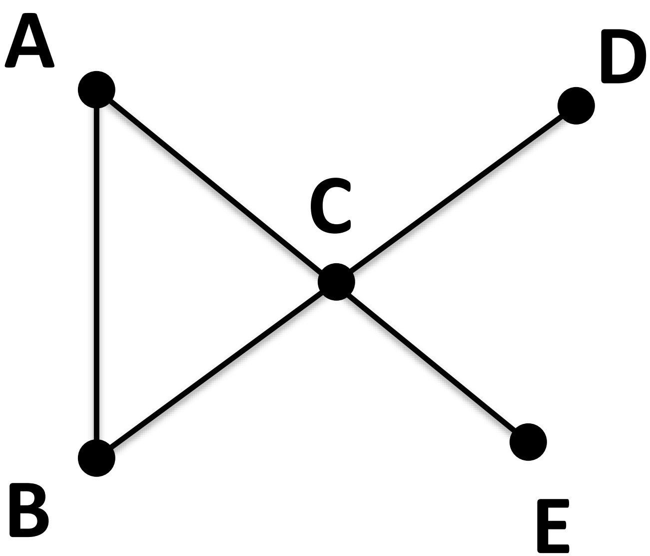
☆ **Examples:**

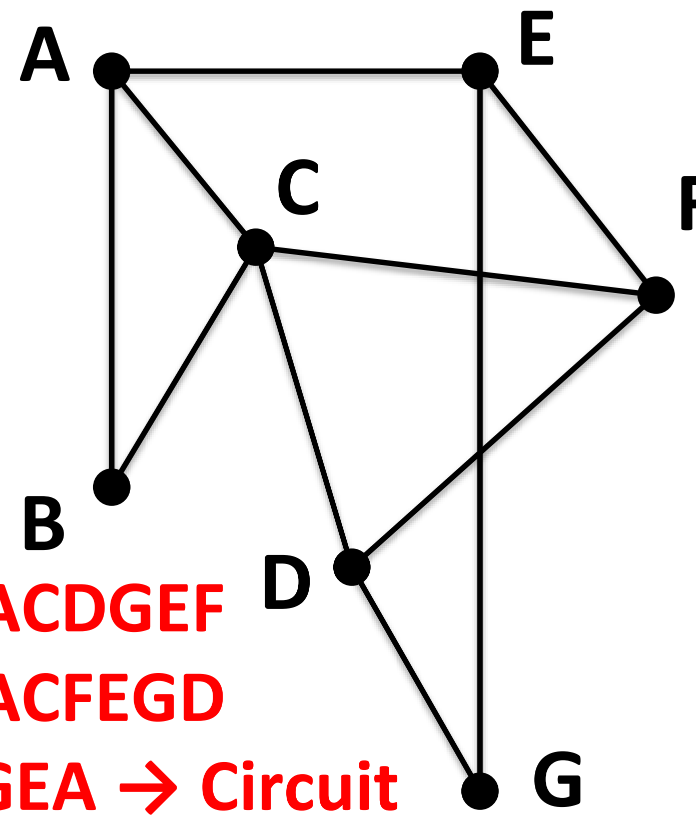
Find a Hamilton Path in each graph shown.

(all vertices once)

Is the path also a Hamilton circuit?

(begins and ends at the same vertex)



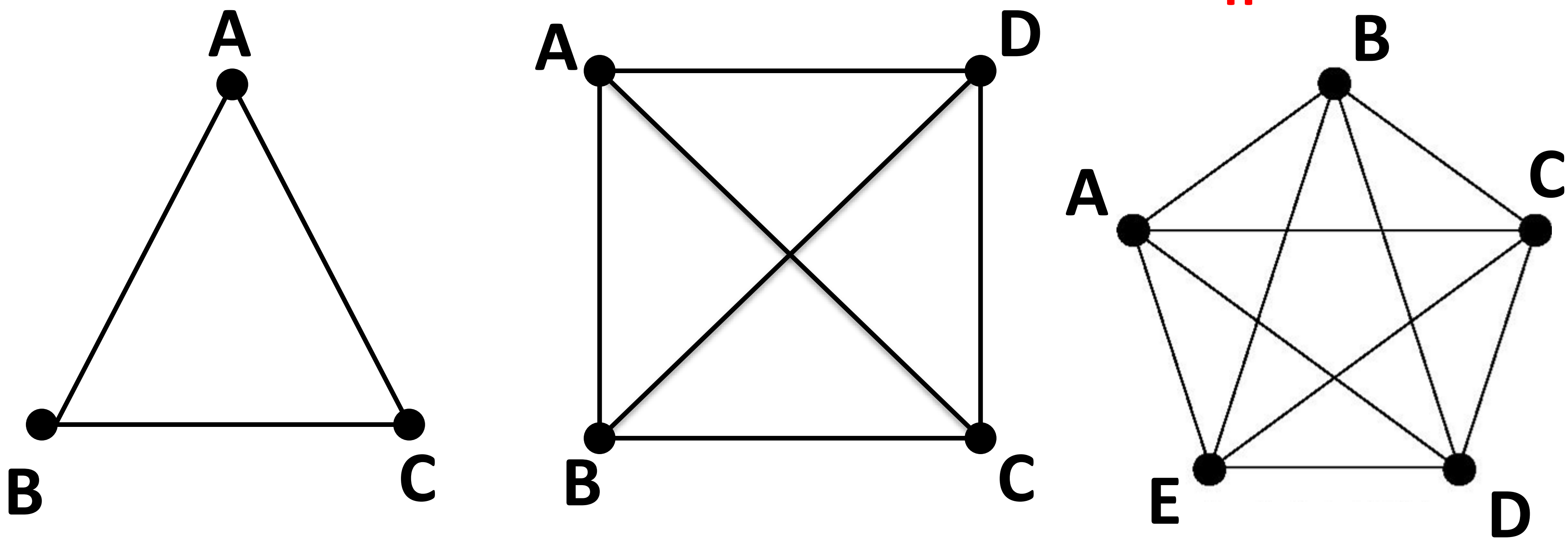


BACDGEF

BACFEGD

ABCFDGEA → Circuit

☆ A **complete graph** is one in which every ___pair___ of ___vertices___ is joined by an ___edge___. A complete graph with *n* vertices is denoted K_n .



Observations:

→ What do you notice about the degree at each vertex in relation to the number of vertices *n*?

→Can you come up with a formula for the number of edges in a complete graph with *n* vertices?

n (vertices)	# of edges
0	0
1	0
2	1
3	3
4	6
5	10
n	?

$$\frac{n(n-1)}{2}$$

☆Often, we will need to find ALL of the Hamilton circuits in a graph (such as in the traveling salesperson example!).

Hints:

✪ Always start with vertex A. (It doesn't matter, but let's just use A to be consistent)

✪ Make a tree-diagram

✪ Do you notice any other pattern?

☆ Example: Find all of the Hamilton circuits in K_4.

☆ Note: Hamilton circuits occur in ___**pairs**___. If you find one Hamilton circuit, ___**reverse**___ the ___**order**___, and you have a new one

☆ What would the tree-diagram for K_5 look like?

Can you figure out the formula for the number of Hamilton Circuits in K_n?

Vertices (n)	# of Hamilton Cir.
3	2
4	6
5	24
6	120
…n	

☆ **The # of Hamilton circuits in K_n:**

K_n has (n-1)•(n-2)•(n-3)•(n-4) .....•(3)•(2)•(1). This number is written **(n – 1)!** and is called (n-1) **factorial**.

5 Vertices:

$= (5 - 1)!$

$= 4!$

$= 4 \cdot 3 \cdot 2 \cdot 1$

$= 24$ Hamilton Circuits

☆ When we assign ____numbers____ to the ____edges____ of a graph, the graph is called a **weighted graph** and the ____numbers____ on the ____edges____ are called **weights**.

☆ The **weight of a path** in a ____weighted____ graph is the ____sum____ of the weights on the ____edges____ of the path.

☆Let’s go back and solve the Traveling Salesperson Example.

☆***Figure 4.7*** A weighted graph of the TSP example. **The weights represent money.**

☆The Brute-Force Algorithm

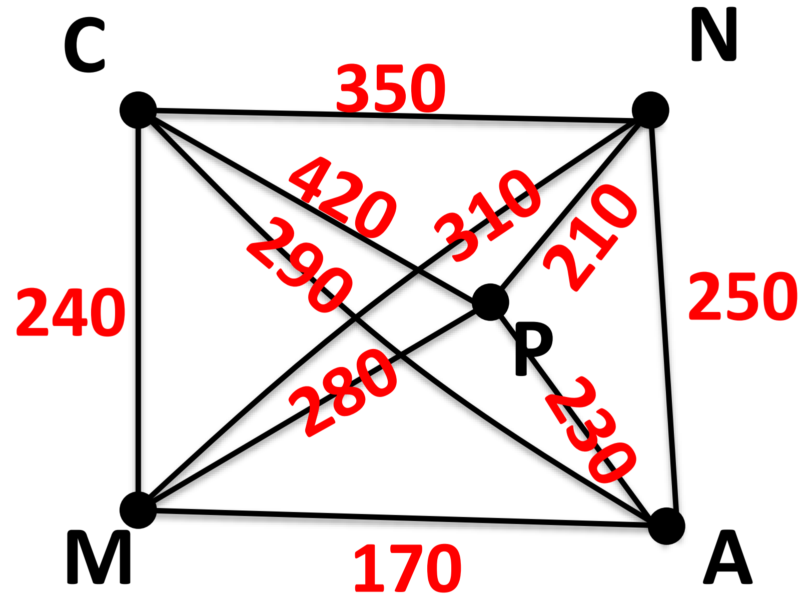
1. List all Hamilton Circuits in the graph

1. Find the weights of each circuit from Step 1

3. The circuits with the smallest weight give the solutions to the TSP.

Circuit	Weight ($)
PACMNP	
PACNMP	
PAMCNP	
PAMNCP	
PANCMP	
PANMCP	
PCAMNP	
PCANMP	
PCMANP	
PCNAMP	
PMACNP	
PMCANP	

Why do we only need to list 12 circuits?



☆Was this method reliable? ___**YES!**_________Was this method fast?___**NO!**________ What if Danielle had to visit 15 cities?!?!?!?

→ Let's say we could examine 1 circuit per minute. How many years would this take?

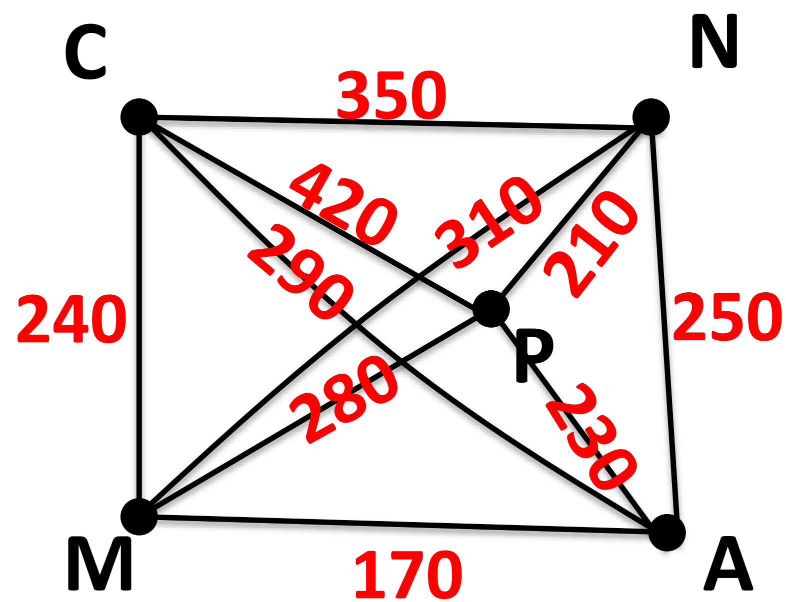
<u>15 Vertices:</u>

= (15 – 1)!
= 14!
= 87,178,291,200 Hamilton Circuits

87,178,291,200**/60** = 1,452,971,520 hours
1,452,971,520**/24** = 60,540,480 Days
60,540,480**/365** = **165,864 YEARS!**

☆The Nearest-Neighbor Algorithm

1. *Start at any vertex x.*
2. *Choose the edge connected to x that has the smallest weight.*
3. *Choose subsequent new vertices as you did in step 2.*
4. *Close the circuit by returning to the starting vertex.*



☆**Example:** Use the Nearest-Neighbor Algorithm to schedule Danielle's trip.

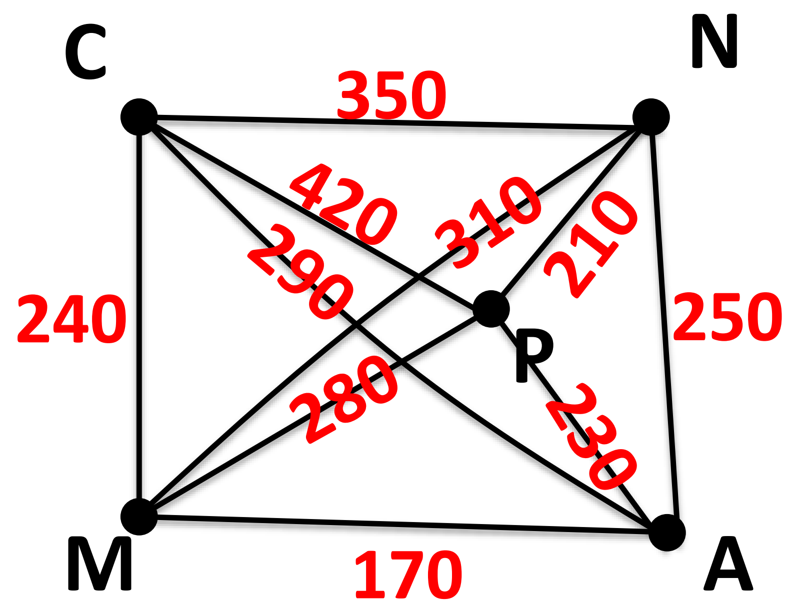
P → N → A → M → C → P

\$210 + \$250 + \$170 + \$240 + \$420 = \$1290

☆ Pros and cons of Nearest-Neighbor compared with Brute-Force Algorithm?

☆The Best-Edge Algorithm

1. *Begin by choosing any edge with the smallest weight.*
2. *Choose any remaining edge in the graph with the smallest weight.*
3. *Keep repeating Step 2; Do not close a circuit until all vertices have been joined.*



☆**Example:** Use the Best-Edge Algorithm to schedule Danielle's trip.

P → N → C → M → A → P

P → A → M → C → N → P

$1200

☆How does this result compare to the Brute-Force Algorithm?

☆Do you think this result is guaranteed every time with this method?