

Objectives

Students will be able to:

- find the **variance** of a data set.
- find the **standard deviation** of a data set.

Variance

Variance is the average squared deviation from the mean of a set of data. It is used to find the **standard deviation**.

So what does this mean....

- ▶ “Deviation”
 - ▶ What are we deviating from?
 - ▶ How can we find how much we’ve “deviated”?
- ▶ “Squared” Deviation
- ▶ “Average” Squared Deviation
 - ▶ What is this telling us to do?

Variance

1. Find the **mean** of the data.

Hint - mean is the average so add up the values and divide by the number of items.

2. Subtract the mean from each value - the result is called the **deviation from the mean**.
3. **Square** each deviation of the mean.
4. Find the **sum** of the squares.
5. **Divide the total** by the number of items.

Variance Formula

The **variance** formula includes the Sigma Notation, Σ , which represents the **sum** of all the items to the right of Sigma.

$$\frac{\sum (x - \mu)^2}{n}$$

Mean is represented by μ and n is the number of items.

The **data point** is represented by x .

In English...

- ▶ Subtract each data point from the mean.
- ▶ Square each of those.
- ▶ Add all of it up.
- ▶ Divide by the # of data points.

- ▶ $x - \mu$

Where x is a data point

Do this for each data point

- ▶ $(x - \mu)^2$

Do this for each data point

- ▶ Add all the $(x - \mu)^2$

- ▶ How many data points are there? DIVIDE!

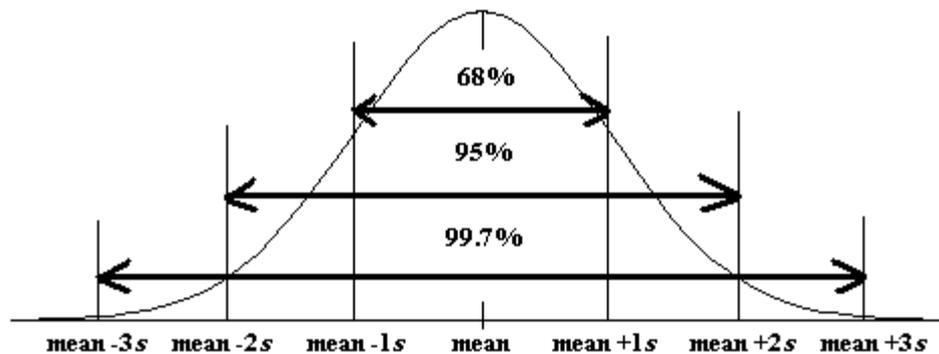
Standard Deviation

Standard Deviation shows the variation in data.

- If the data is close together, the standard deviation will be **small**.
- If the data is spread out, the standard deviation will be **large**.

Standard Deviation is often denoted by the lowercase Greek letter sigma σ .

The **bell curve** which represents a normal distribution of data shows what standard deviation represents.



One standard deviation away from the mean (μ) in either direction on the horizontal axis accounts for around **68 percent** of the data. Two standard deviations away from the mean accounts for roughly **95 percent** of the data with three standard deviations representing about **99 percent** of the data.

Standard Deviation

Find the **variance**.

- a) Find the **mean** of the data.
- b) Subtract the mean from each value.
- c) Square each deviation of the mean.
- d) Find the sum of the squares.
- e) Divide the total by the number of items.

Take the square root of the variance.

Standard Deviation Formula

The standard deviation formula can be represented using Sigma Notation:

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

Notice the standard deviation formula is the square root of the variance.

Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

1) Find the **mean**: $(92+88+80+68+52)/5 = 76$.

2) Find the **deviation from the mean**:

$$92-76=16$$

$$88-76=12$$

$$80-76=4$$

$$68-76=-8$$

$$52-76=-24$$

Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

3) Square the deviation from the mean:

$$(16)^2 = 256$$

$$(12)^2 = 144$$

$$(4)^2 = 16$$

$$(-8)^2 = 64$$

$$(-24)^2 = 576$$

Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

4) Find the sum of the squares of the deviation from the mean:

$$256 + 144 + 16 + 64 + 576 = 1056$$

5) Divide by the number of data items to find the **variance**:

$$1056 / 5 = 211.2$$

Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

6) Find the square root of the variance: $\sqrt{211.2} = 14.53$

The **standard deviation** of the test scores is **14.53**.

Standard Deviation

A different math class took the same test with these five test scores: 92,92,92,52,52.

Find the **standard deviation** for this class.

Hint:

1. Find the **mean** of the data.
2. Subtract the mean from each value - called the **deviation from the mean**.
3. Square each deviation of the mean.
4. Find the sum of the squares.
5. Divide the total by the number of items - result is the **variance**.
6. Take the square root of the variance - result is the **standard deviation**.

Solve:

A different math class took the same test with these five test scores: 92,92,92,52,52.

Find the **standard deviation** for this class.

Answer Now

The math test scores of five students are: 92,92,92,52 and 52.

1) Find the **mean**: $(92+92+92+52+52)/5 = 76$

2) Find the deviation from the mean:

$$92-76=16 \quad 92-76=16 \quad 92-76=16$$

$$52-76= -24 \quad 52-76= -24$$

3) Square the deviation from the mean:

$$(16)^2 = 256 \quad (16)^2 = 256 \quad (16)^2 = 256$$

$$(-24)^2 = 576 \quad (-24)^2 = 576$$

4) Find the sum of the squares:

$$256+256+256+576+576= 1920$$

The math test scores of five students are: 92, 92, 92, 52 and 52.

5) Divide the sum of the squares by the number of items :

$$1920/5 = 384 \text{ variance}$$

6) Find the square root of the variance:

$$\sqrt{384} = 19.6$$

The **standard deviation** of the second set of test scores is 19.6.

Analyzing the data:

Consider both sets of scores. Both classes have the **same mean, 76**. However, each class does not have the same scores.

We use the standard deviation to show the **variation** in the scores.

With a standard variation of 14.53 for the first class and 19.6 for the second class, what does this tell us?

Answer Now

Analyzing the data:

Class A: 92,88,80,68,52

Class B: 92,92,92,52,52

With a standard variation of 14.53 for the first class and 19.6 for the second class, the scores from the second class would be **more** spread out than the scores in the first class.

Analyzing the data:

Class A: 92,88,80,68,52

Class B: 92,92,92,52,52

Class C: 77,76,76,76,75

Estimate the standard deviation for Class C.

- a) Standard deviation will be less than 14.53.
- b) Standard deviation will be greater than 19.6.
- c) Standard deviation will be between 14.53 and 19.6.
- d) Can not make an estimate of the standard deviation.

Answer Now

Analyzing the data:

Class A: 92,88,80,68,52

Class B: 92,92,92,52,52

Class C: 77,76,76,76,75

Estimate the standard deviation for Class C.

- a) Standard deviation will be less than 14.53.
- b) Standard deviation will be greater than 19.6.
- c) Standard deviation will be between 14.53 and 19.6
- d) Can not make an estimate if the standard deviation.

Answer: A

The scores in class C have the same mean of 76 as the other two classes. However, the scores in Class C are all much closer to the mean than the other classes so the standard deviation will be smaller than for the other classes.

Summary:

As we have seen, **standard deviation** measures the dispersion of data.

The greater the value of the **standard deviation**, the further the data tend to be dispersed from the mean.

HOMEWORK:

- ▶ Complete the standard deviation worksheet on Blackboard under assignments.
- ▶ Make sure **all work** is shown in order to receive credit.
- ▶ Have a great weekend!