Warm Up

1. Find the MOE for a 95% confidence level with a standard deviation of 5.2 and a sample size of 30.

2. A news reporter reports the results of a survey and states that 45% of those surveyed responded "yes" with a MOE of plus/minus 5%. Explain what that means.

3. Find the MOE for a 90% confidence level with a standard deviation of 1.5 and a sample size of 50.

More Practice

1. With a confidence level of 90%, mean of 57.2, standard deviation of 7.1, and sample size of 50, determine the confidence interval. Make a number line representation.

2. Use the confidence interval (12, 14.8) to determine the margin of error and sample mean.

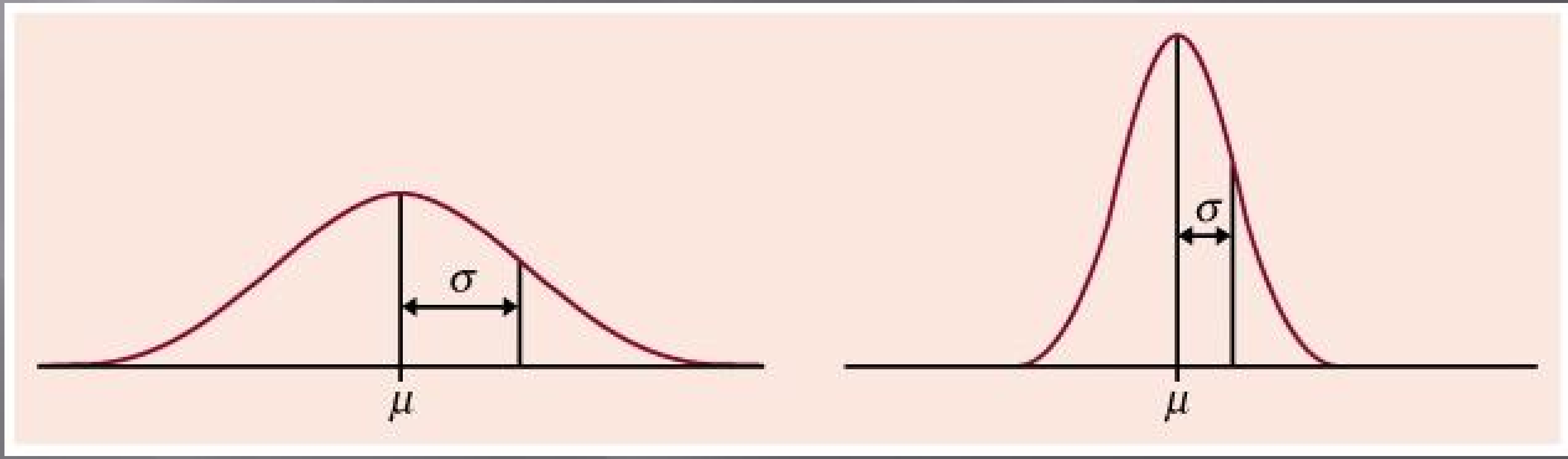
3. Determine the minimum required sample size if you want to be 90% confident that the sample mean is within 1 unit of the population mean given the standard deviation is 6.8.

NORMAL DISTRIBUTIONS THE EMPIRICAL RULE AND Z-SCORE

OBJECTIVE: Use the empirical rule (68-95-99.7 rule) to analyze data

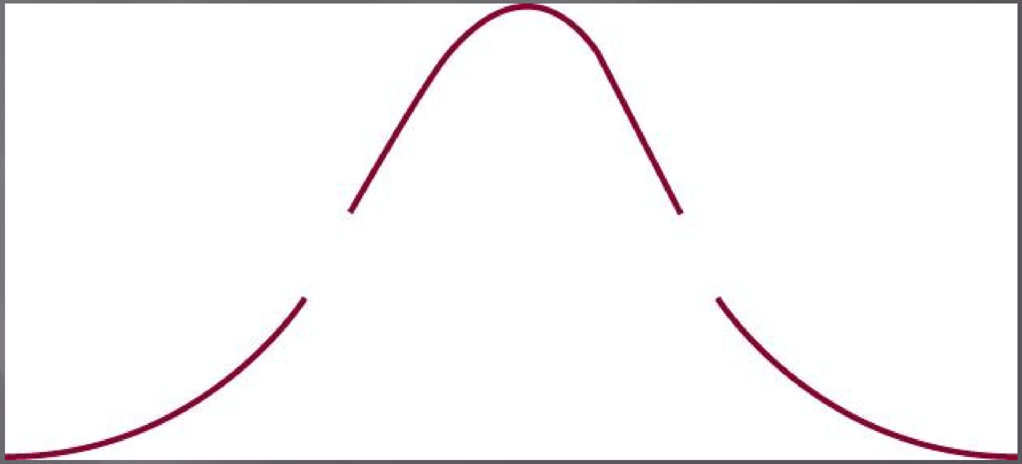
Normal distributions: N (μ, σ)

- Symmetric, single peaked and bell shaped.
- Center of the curve are μ and M.
- Mean, median, & mode are the same
- Standard deviation σ controls the spread of the curve.



Normal distributions: N (μ, σ)

- Inflection points: points where change of curvature takes place.
- Located a distance σ (standard deviation) on either side of μ (the mean).
- We will draw dotted lines at these inflection points!



Normal curves are a good description of some real data:

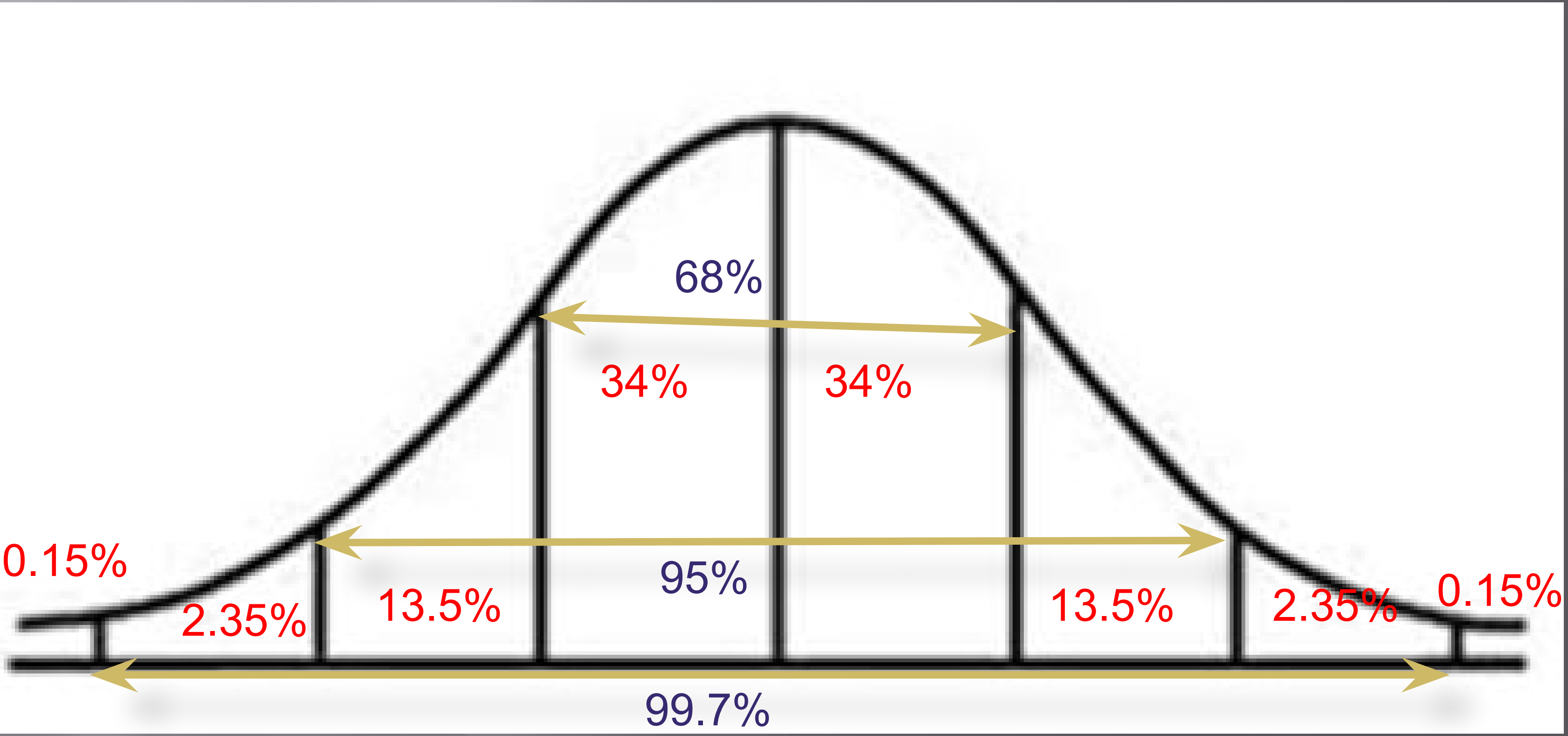
- test scores (SAT, ACT, IQ)
- biological measurements
- also approximate chance outcomes like tossing coins

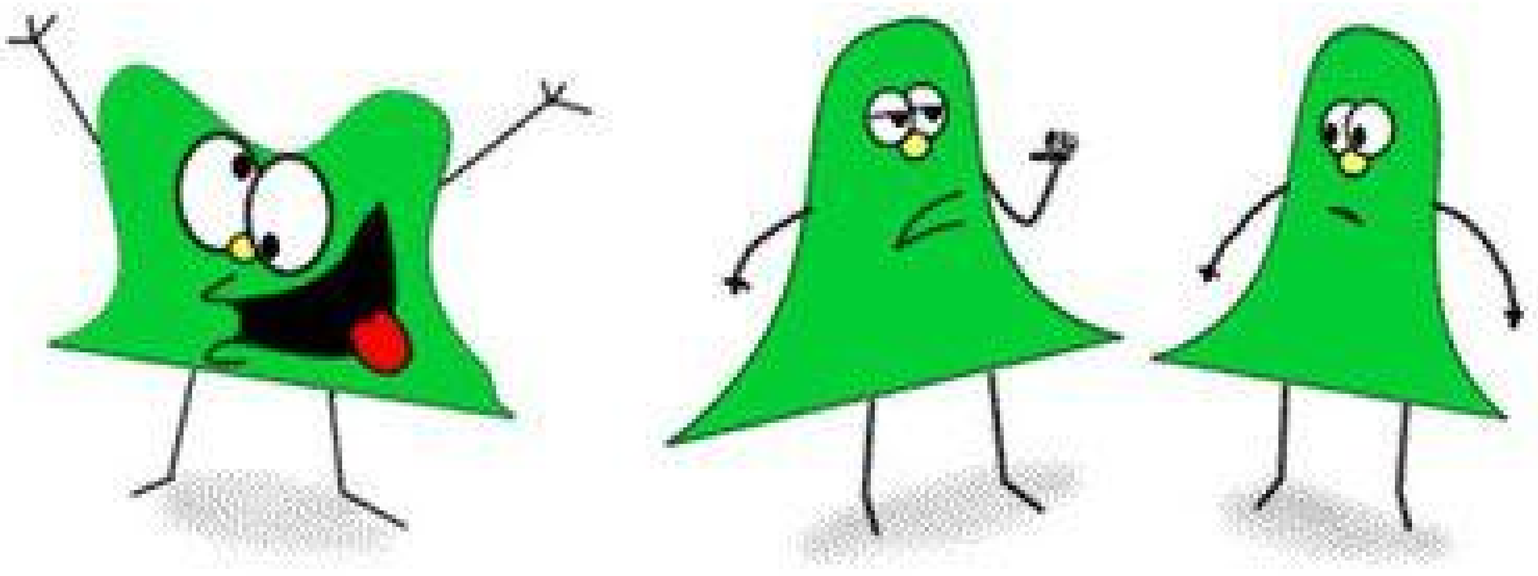
The Empirical rule (68-95-99.7 rule)

In the normal dist. with mean μ and standard deviation σ.

- 68% of the observations fall within 1σ of the mean.
- 95% of the observations fall within 2σ of the mean.
- 99.7% of the observations fall within 3σ of the mean.

Behold the normal curve



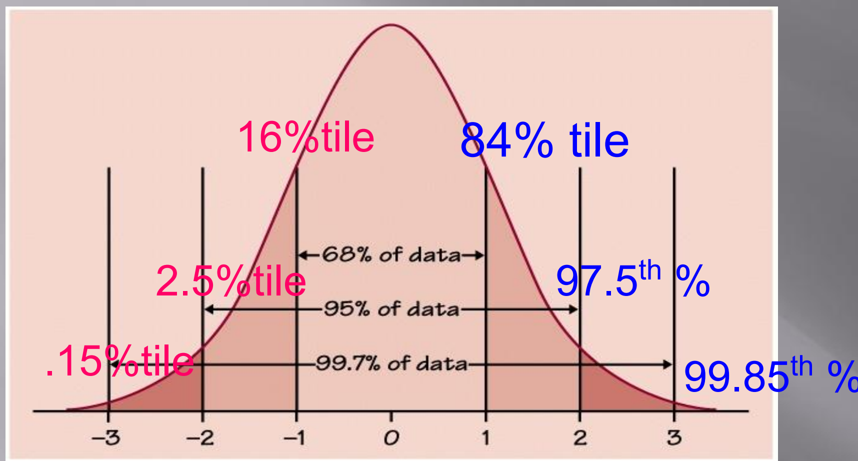


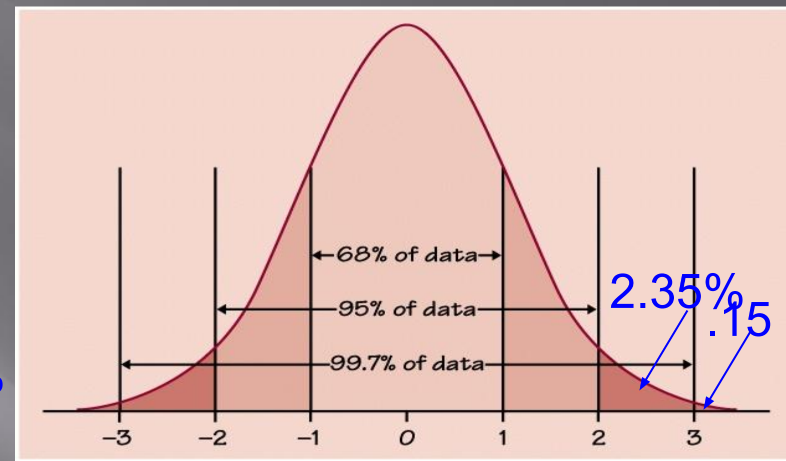
"KEEP YOUR EYE ON THAT GUY, TOM. HES NOT, YOU KNOW...NORMAL!"

Example1: Men's Heights

The distribution of adult American men is approximately normal with mean 70 inches and standard deviation 2.5 inches. Draw the curve and mark points of inflection.

Recall: mean 70 in. and standard deviation 2.5 in.





62.5 65 67.5 70 72.5 75 77.5

a) What percent of men are taller than 75 inches?

75 is two standard dev. above the mean.

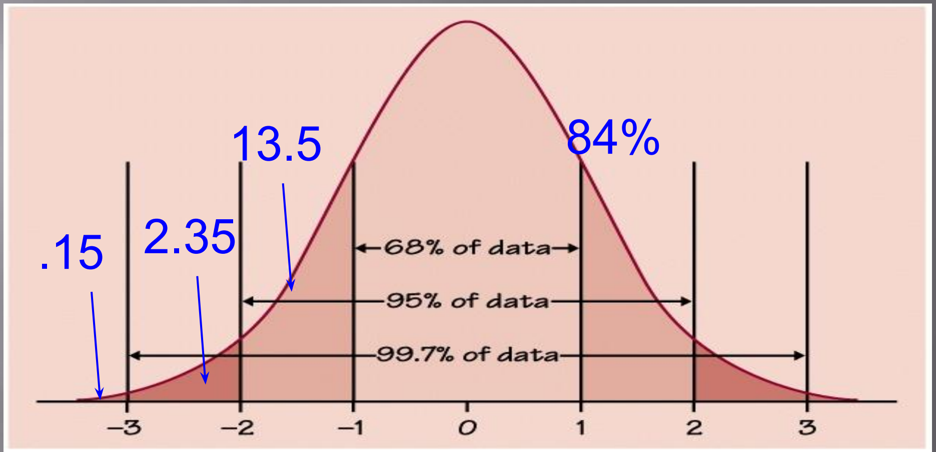
2.35 + .15 =2.5

b) Between what heights do the middle 95% of men fall?

Between 65 and 75 inches

±

mean 70 in. and standard deviation 2.5 in.



62.5 65 67.5 70 72.5 75 77.5

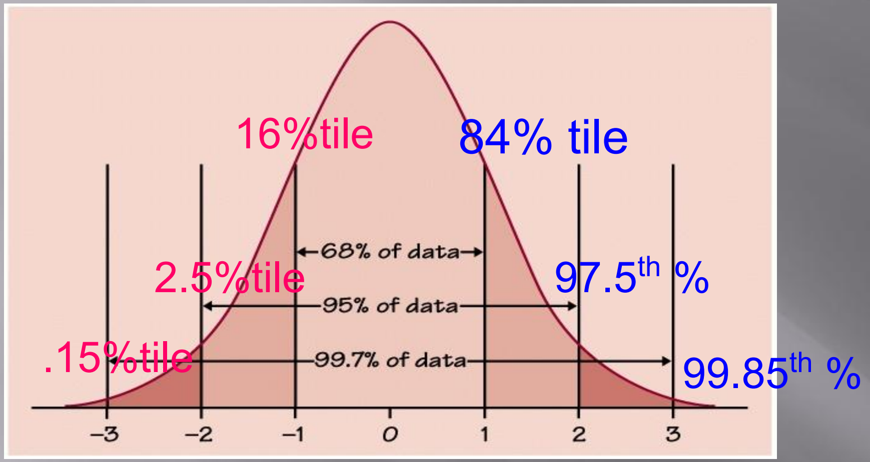
c) What percent of men are shorter than 67.5 inches?

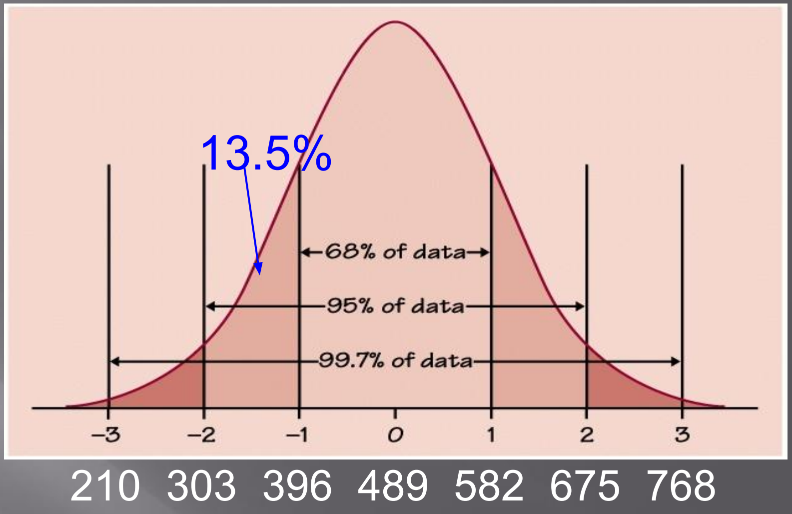
16.0%

Example 2: SAT Verbal Scores

SAT verbal scores are normally distributed with a mean of 489 and a standard deviation of 93.

Recall: mean 489 in. and standard deviation 93 in.



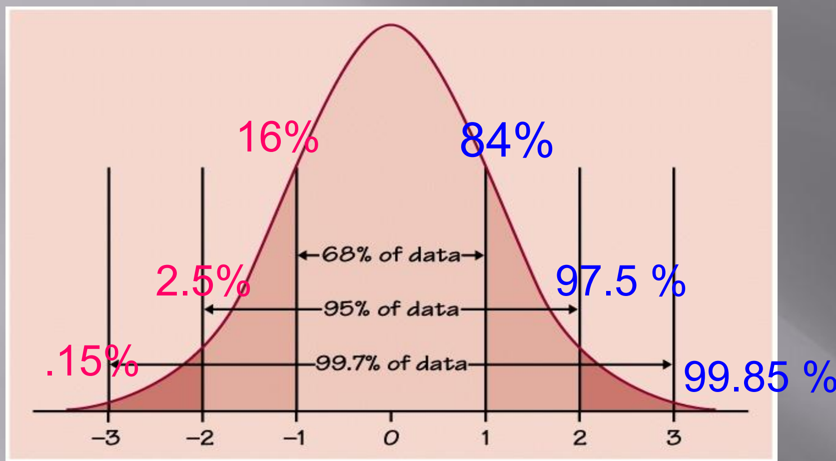


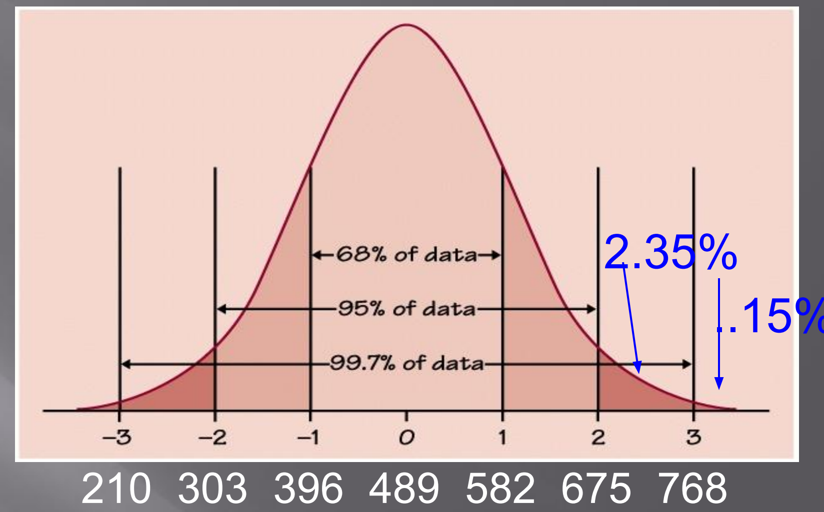
a) What percentage lie between 303 and 582?

303 is two standard dev. below the mean. & 582 is one std. dev above

68 + 13.5 = 81.5%

Recall: mean 489 in. and standard deviation 93 in.





b) What percentage is above 675?

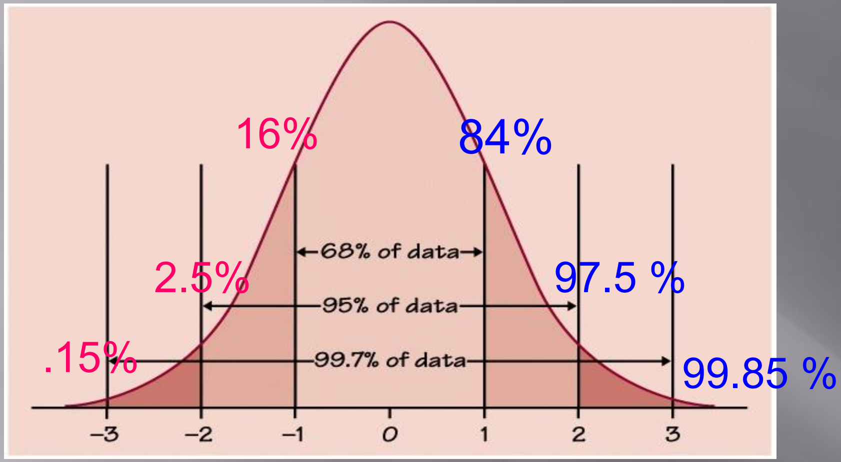
675 is two standard dev. above the mean.

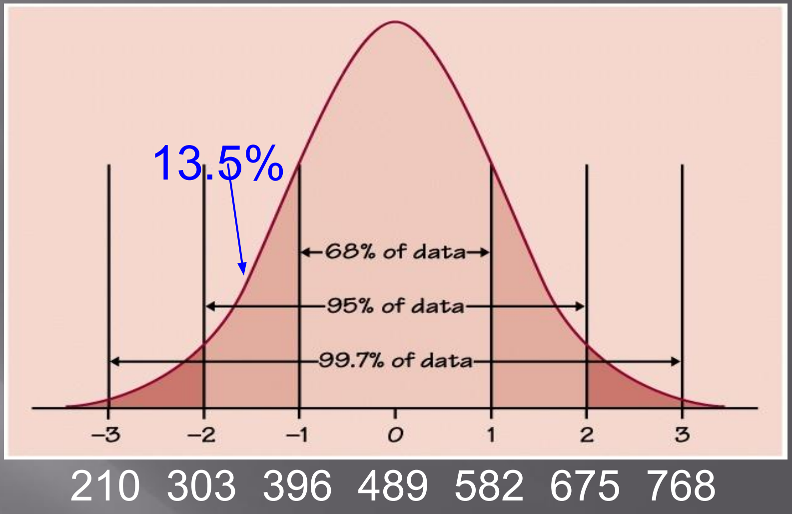
2.35 +.15 = 2.5%

c) **What percentage is below 675**

100 - 2.5 = 97.5

Recall: mean 489 in. and standard deviation 93 in.





d) If 3,500 students took the SAT verbal test, about how many received between 396 and 675 points?

68% + 13.5% = 81.5% fall within this range.

3,500 * 81.5% = 2853

z-scores

When a set of data values are normally distributed, we can standardize each score by converting it into a ***z*-score**.

***z*-scores** make it easier to compare data values measured on different scales.

z-scores

A ***z*-score** reflects how many standard deviations **above** or **below** the mean a raw score is.

The ***z*-score** is **positive** if the data value lies **above** the mean and **negative** if the data value lies **below** the mean.

z-score formula

$$z = \frac{x - \mu}{\sigma}$$

Where x represents an element of the data set, the mean is represented by μ and standard deviation by σ.

Example 3: Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her *z*-score?

Answer Now

Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her z-score?

$$z = \frac{700 - 500}{100} = 2$$

Her z-score would be 2 which means her score is two standard deviations above the mean.

Analyzing the data

- A set of math test scores has a mean of 70 and a standard deviation of 8.
- A set of English test scores has a mean of 74 and a standard deviation of 16.

For which test would a score of 78 have a higher standing?

Answer Now

Analyzing the data

A set of math test scores has a mean of 70 and a standard deviation of 8.
A set of English test scores has a mean of 74 and a standard deviation of 16.
For which test would a score of 78 have a higher standing?

To solve: Find the z-score for each test.

$$\text{math } z\text{-score} = \frac{78-70}{8} = 1$$

$$\text{English } z\text{-score} = \frac{78-74}{16} = .25$$

The math score would have the highest standing since it is 1 standard deviation above the mean while the English score is only .25 standard deviation above the mean.

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Answer Now

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Using the formula for z-scores: $z = \dfrac{x - \mu}{\sigma}$

$$1.5 = \frac{x - 23}{2} \qquad 3 = x - 23 \qquad x = 26$$

The Toyota Camry would be expected to use 26 mpg of gasoline.

Analyzing the data

A group of data with normal distribution has a mean of 45. If one element of the data is 60, will the z-score be positive or negative?

Answer Now

Analyzing the data

A group of data with normal distribution has a mean of 45. If one element of the data is 60, will the z-score be positive or negative?

The z-score must be positive since the element of the data set is above the mean.