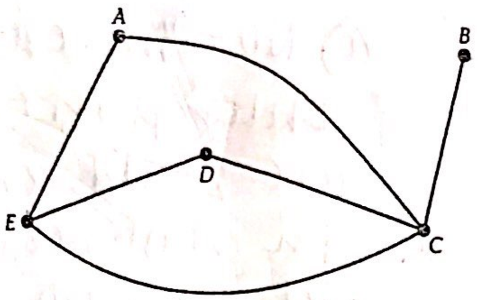
Unit 2 – Graph Theory Review

Name Key

(1)



a. Is this graph connected? Explain why or why not. Yes – it is possible to travel from any vertex to any other vertex.

b. Is this graph complete? Explain why or why not. NO - Not all vertices are connected by an edge

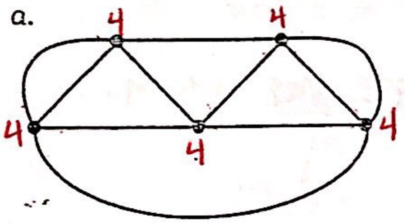
c. Name two vertices that are adjacent to vertex E. D and A

d. Name a path from B to E of length 3. BCDE or BCAE

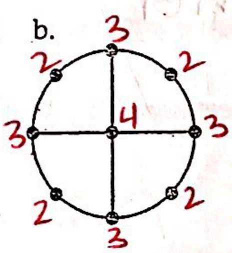
e. What is the degree of vertex C? 4

(2) State whether each graph has an Euler circuit, an Euler path, or neither. Explain why.

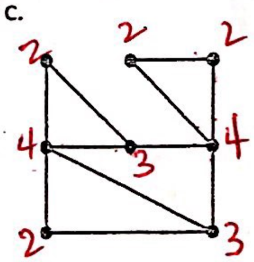
a.



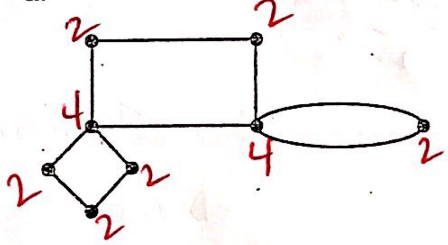
b.



c.



d.



a) Euler Circuit All Even vertices

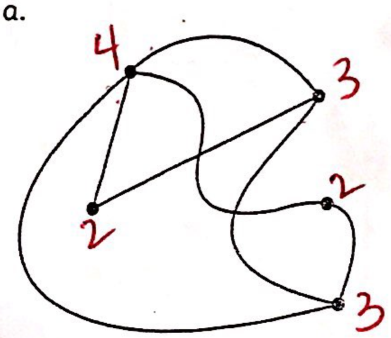
b) Neither 4 odd vertices

c) Euler path 2 odd vertices.

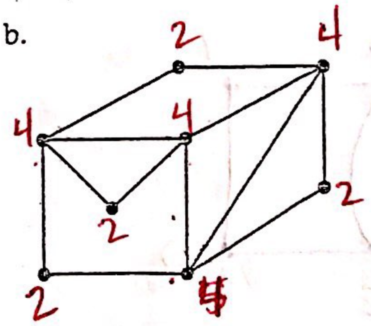
d) Euler circuit All Even vertices

(3) Tell whether the following graphs have an Euler circuit, an Euler path, or neither. Explain your answers.

a.



b.



a) Euler path 2 odds

b) Euler circuit All even vertices.

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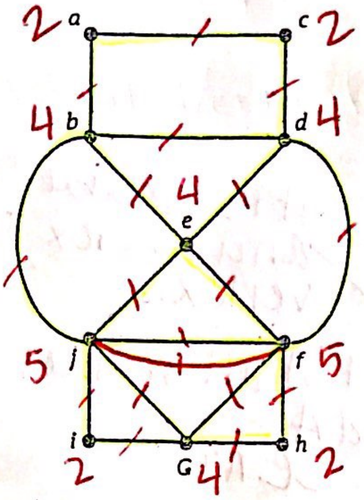
4) The street network of a city can be modeled with a graph in which the vertices represent the street corners, and the edges represent the streets. Suppose you are the city street inspector and it is desirable to minimize time and cost by not inspecting the same street more than once.



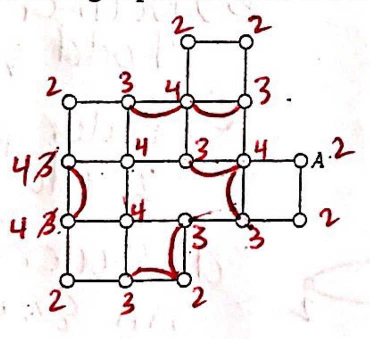
a) NO - There is an Euler path (b/c there are 2 odd vertices) but you must begin & end at the odds to go over all edges.

a. In this graph of the city, is it possible to begin at the garage (G) and inspect each street only once? Will you be back at the garage at the end of the inspection?

b. Find a route that inspects all streets, repeats the least number of edges possible, and returns to the garage. Need to Eulerize (get rid of odds)

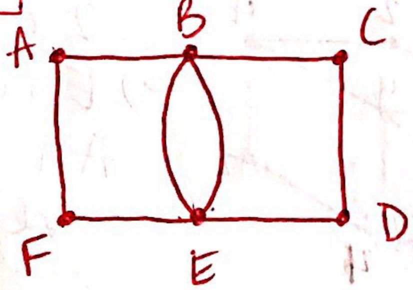
GijbacdbedfejfjGfhG

5) Consider the following representation of a street network. Eulerize the graph.



6) Draw a graph with six vertices and eight edges so that the graph has an Euler circuit.

All Even vertices



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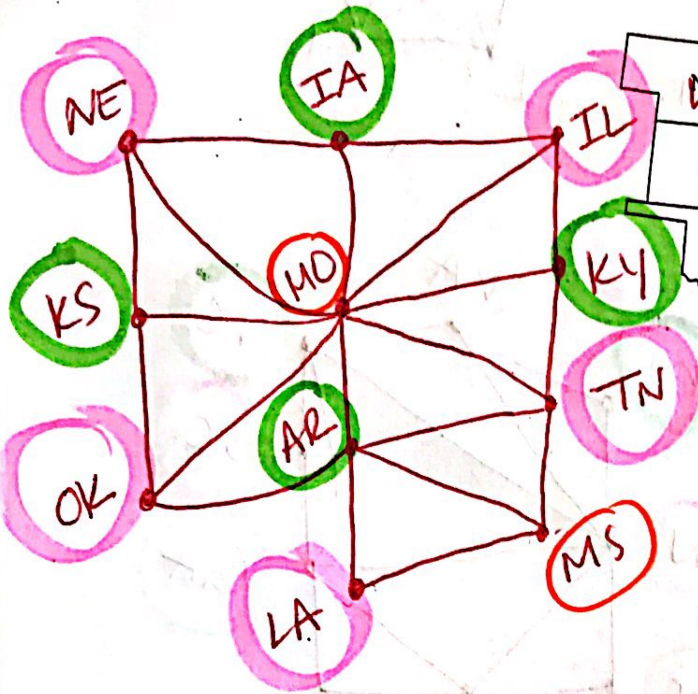
7) Draw graphs to represent the following maps. Color the graphs. What is the minimum number of colors needed to color each map?

b.



Pink: NE, IL, TN, LA, OK

Green: IA, KS, AR, KY

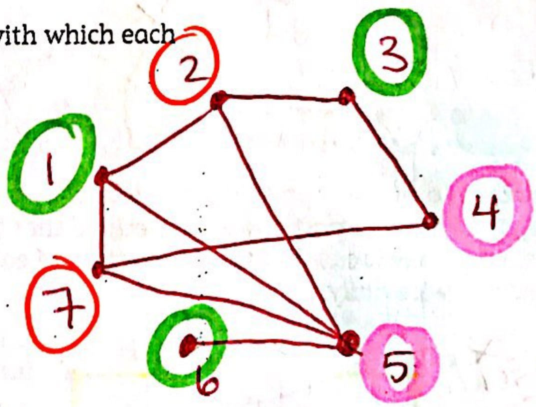
Orange: MO, MS

Answers vary.

★ CAN be done w/ 3 colors ★

8) Following is a list of chemicals and the chemicals with which each cannot be stored.

Chemicals	Cannot Be Stored With
1	2, 5, 7 ✓
2	1, 3, 5 ✓
3	2, 4 ✓
4	3, 7 ✓
5	1, 2, 6, 7 ✓
6	5 ✓
7	1, 4, 5 ✓



Orange: 2, 7

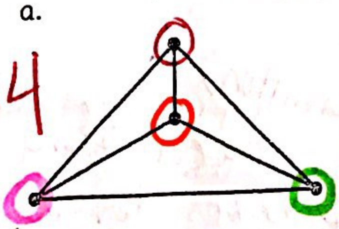
Green: 1, 3, 6

Pink: 4, 5

How many different storage facilities are necessary in order to keep all seven chemicals?

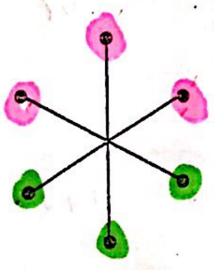
9) Find the chromatic number for each of the following graphs.

a.



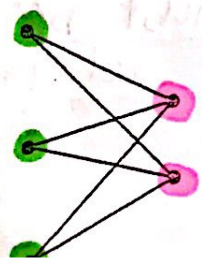
4

b.



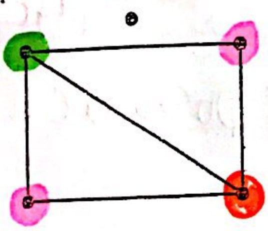
2

c.



2

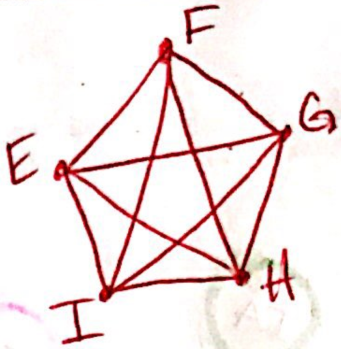
d.



3

(10)

a. Draw a K_5 graph. Label the vertices E, F, G, H, and I.



b. Deg(G) = ___?___. 4

c. Does the graph have an Euler circuit? Explain.

yes. All vertices are even.

(11)

a. Represent the following map with a graph.

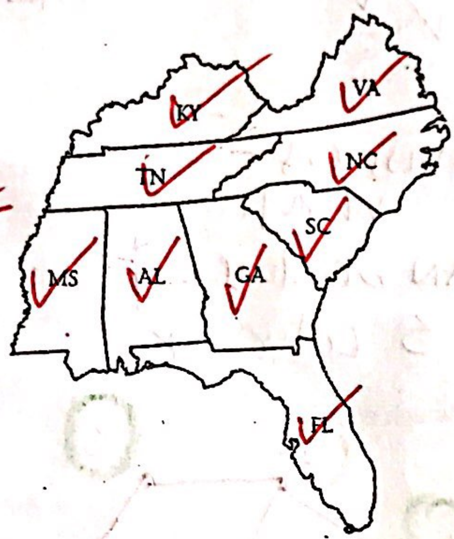
b. Color your graph.

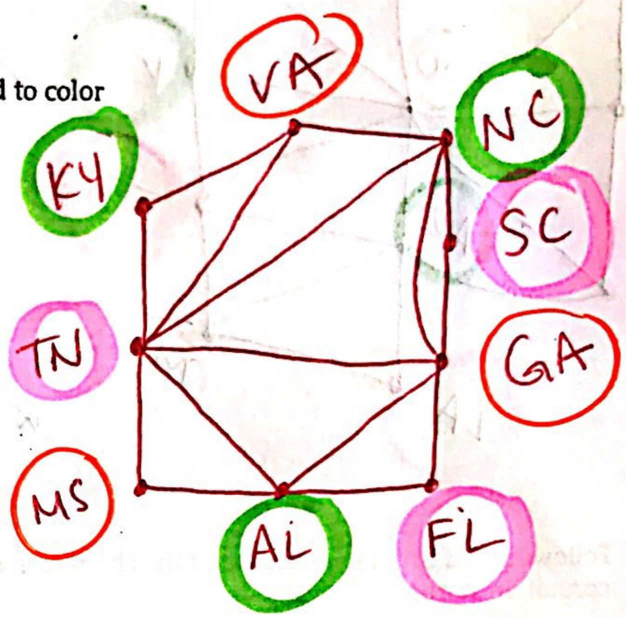
c. What is the minimum number of colors needed to color the map?

Green
KY
NC
AL

Orange
VA
MS

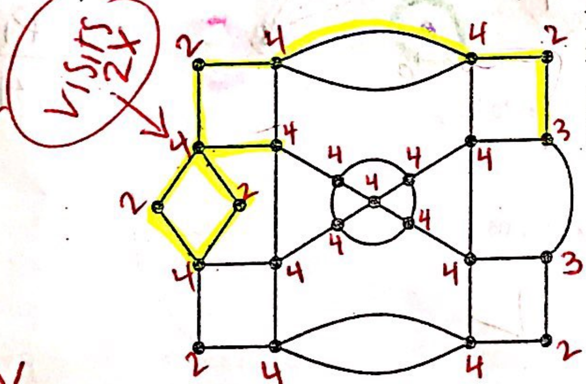
Pink
TN
FL
SC





Following is a multigraph that represents the downtown area of a small city. The local post office has decided that the mail drop boxes, which are located at the intersection of each street, must be monitored twice daily.

Euler (Edges)
(2 odds)



Is it possible for the local street inspector to begin at an intersection and inspect each street exactly once? Yes

Is it possible for the inspector to finish her route at the same intersection from which she began? Explain why or why not.

NO - You need zero odds for a circuit.

(Hamilton) - All vertices only once

a. Is it possible to find a circuit that begins and ends at the same intersection and visits each drop box exactly once? NO - You would have to visit drop boxes several times to get to others.

b. If not, is there a path that begins at one drop box, visits each drop box exactly once, and ends at a different drop box? NO

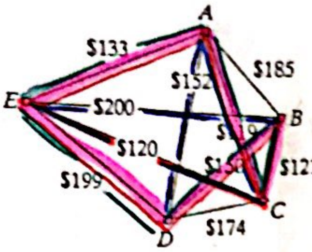
c. If either route exists, copy the figure onto your paper and darken the edges of your proposed route.

Not possible.

(34)

Scanned with CamScanner

(12) For the weighted graph shown in the figure, (i) find the indicated circuit, and (ii) give its cost. (This is the graph discussed in Example 6.7.)



(a) The nearest-neighbor circuit for starting vertex B

(b) The nearest-neighbor circuit for starting vertex C

(c) The nearest-neighbor circuit for starting vertex D

(d) The nearest-neighbor circuit for starting vertex E

e) The best edge algorithm

a) BCAEDB $722 PINK

b) CAEDBC $722 Red

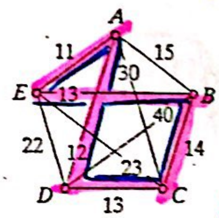
c) DBCAED $722 green

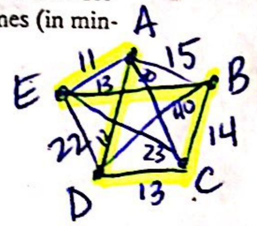
d) ECADBE $741 Blue

e) $741 →

119
then 120
cant do 121
cant do 133
then 150
then 152
cant do 174, 185
199
Then 200

(13) A delivery service must deliver packages at Buckman (B), Chatfield (C), Dayton (D), and Evansville (E), and then return to Arlington (A), the home base. The following graph shows the estimated travel times (in minutes) between the cities.





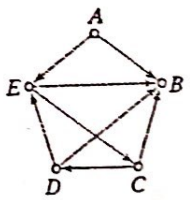
AEBCDA

(a) Find the nearest-neighbor circuit for starting vertex A. What is the total travel time of this trip? 63

(b) Find the nearest-neighbor circuit for starting vertex D. Write the answer as it would be traveled if starting and ending at A. DAEBCD 63

c) Find a circuit using the best edge algorithm. Same 63

(14) Consider the digraph shown in the following figure.



(a) Find a path from vertex A to vertex D. AECD

(b) Explain why the path you found in (a) is the only possible path from vertex A to vertex D. A→B will get you stuck there

(c) Find a cycle in the digraph. → ECDE

(d) Explain why vertex A cannot be part of a cycle. There's no way back to A if you leave

(e) Explain why vertex B cannot be part of a cycle. There's no way out of B.

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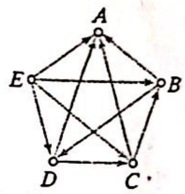
(15) The digraph in the following figure is an example of a *tournament* digraph. In this example the vertices of the digraph represent five volleyball teams in a round-robin tournament (i.e., every team plays every other team). An arc XY represents the fact that X defeated Y in the tournament.

(*Note: There are no ties in volleyball.*)



(a) Which team won the tournament? Explain. E

(b) Which team came in last in the tournament? Explain. A

BEAT

Team	A	B	C	D	E	SUM
A	0	0	0	0	0	0
B	2	0	1	1	0	4
C	2	1	0	1	0	4
D	2	1	1	0	0	4
E	3	2	2	2	0	9

35