ICM Honors Name___________________________________

Unit 4 Quiz 2 REVIEW

Date________________ Period____

Solve each related rate problem.

1) A hypothetical square grows so that the length of its sides are increasing at a rate of 4 m/min. How fast is the area of the square increasing when the sides are 15 m each?

2) A hypothetical square shrinks so that the length of its sides are changing at a rate of -7 m/min. At what rate is the area of the square changing when the sides are 10 m each?

3) A hypothetical square shrinks at a rate of 16 m^2/min. At what rate are the sides of the square changing when the sides are 10 m each?

4) A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 7 ft/sec. How fast is the area taken up by the crowd increasing when the radius is 14 ft?

5) Water slowly evaporates from a circular shaped puddle. The area of the puddle decreases at a rate of 49π in^2/hr. Assuming the puddle retains its circular shape, at what rate is the radius of the puddle changing when the radius is 13 in?

6) A perfect cube shaped ice cube melts at a rate of 27 mm^3/sec. Assume that the block retains its cube shape as it melts. At what rate are the sides of the ice cube changing when the sides are 7 mm each?

7) A spherical snowball is rolled in fresh snow, causing it grow at a rate of 36π in^3/sec. How fast is the radius of the snowball increasing when the radius is 8 in?

8) A spherical balloon is deflated so that its radius decreases at a rate of 3 cm/sec. At what rate is the volume of the balloon changing when the radius is 9 cm?

9) A 10 ft ladder is leaning against a wall and sliding towards the floor. The foot of the ladder is sliding away from the base of the wall at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall when the top of the ladder is 8 ft from the ground?

10) An observer stands 2400 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 600 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 700 ft from the ground?

Evaluate each indefinite integral.

11) $\int (-18x^5 + 15x^4 + 15x^2)\, dx$

12) $\int (10x^4 + 8x)\, dx$

13) $\int (30x^5 + 15x^4 + 4x)\, dx$

14) $\int (8x - x^{-2} - 9x^{-4})\, dx$

15) $\int \left(12x^2 - \frac{12}{x^5}\right) dx$

16) $\int 6x^5 \, dx$

17) $\int -\frac{10x^{\frac{1}{4}}}{4} \, dx$

18) $\int \left(10x^4 + \frac{25x^{\frac{2}{3}}}{3} + \frac{16x^{\frac{1}{3}}}{3}\right) dx$

19) $\int \frac{25\sqrt[4]{x}}{4} \, dx$

20) $\int \left(25x^4 + 8x + 4\sqrt[3]{x}\right) dx$

21) $\int 4x\left(6x^4 + 5x^3 + 2\right) dx$

22) $\int 2x^3\left(-3x^2 + 10x + 10\right) dx$

23) $\int \frac{-2x^2 - 2x - 9}{x^4} \, dx$

24) $\int \frac{-x^3 + 9x - 16}{x^5} \, dx$

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Unit 4 Quiz 2 REVIEW

Date________________ Period____

Solve each related rate problem.

1) A hypothetical square grows so that the length of its sides are increasing at a rate of 4 m/min. How fast is the area of the square increasing when the sides are 15 m each?

A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{ds}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{s=15}$

$$\left.\frac{dA}{dt}\right|_{s=15} = 2s \cdot \frac{ds}{dt} = 120 \text{ m}^2\text{/min}$$

2) A hypothetical square shrinks so that the length of its sides are changing at a rate of −7 m/min. At what rate is the area of the square changing when the sides are 10 m each?

A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{ds}{dt} = -7$ Find: $\left.\frac{dA}{dt}\right|_{s=10}$

$$\left.\frac{dA}{dt}\right|_{s=10} = 2s \cdot \frac{ds}{dt} = -140 \text{ m}^2\text{/min}$$

3) A hypothetical square shrinks at a rate of 16 m²/min. At what rate are the sides of the square changing when the sides are 10 m each?

A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{dA}{dt} = -16$ Find: $\left.\frac{ds}{dt}\right|_{s=10}$

$$\left.\frac{ds}{dt}\right|_{s=10} = \frac{1}{2s} \cdot \frac{dA}{dt} = -\frac{4}{5} \text{ m/min}$$

4) A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 7 ft/sec. How fast is the area taken up by the crowd increasing when the radius is 14 ft?

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 7$ Find: $\left.\frac{dA}{dt}\right|_{r=14}$

$$\left.\frac{dA}{dt}\right|_{r=14} = 2\pi r \cdot \frac{dr}{dt} = 196\pi \text{ ft}^2\text{/sec}$$

5) Water slowly evaporates from a circular shaped puddle. The area of the puddle decreases at a rate of 49π in²/hr. Assuming the puddle retains its circular shape, at what rate is the radius of the puddle changing when the radius is 13 in?

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = -49\pi$ Find: $\left.\frac{dr}{dt}\right|_{r = 13}$

$$\left.\frac{dr}{dt}\right|_{r = 13} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = -\frac{49}{26} \text{ in/hr}$$

6) A perfect cube shaped ice cube melts at a rate of 27 mm³/sec. Assume that the block retains its cube shape as it melts. At what rate are the sides of the ice cube changing when the sides are 7 mm each?

V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{dV}{dt} = -27$ Find: $\left.\frac{ds}{dt}\right|_{s = 7}$

$$\left.\frac{ds}{dt}\right|_{s = 7} = \frac{1}{3s^2} \cdot \frac{dV}{dt} = -\frac{9}{49} \text{ mm/sec}$$

7) A spherical snowball is rolled in fresh snow, causing it grow at a rate of 36π in³/sec. How fast is the radius of the snowball increasing when the radius is 8 in?

V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = 36\pi$ Find: $\left.\frac{dr}{dt}\right|_{r = 8}$

$$\left.\frac{dr}{dt}\right|_{r = 8} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{9}{64} \text{ in/sec}$$

8) A spherical balloon is deflated so that its radius decreases at a rate of 3 cm/sec. At what rate is the volume of the balloon changing when the radius is 9 cm?

V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = -3$ Find: $\left.\frac{dV}{dt}\right|_{r = 9}$

$$\left.\frac{dV}{dt}\right|_{r = 9} = 4\pi r^2 \cdot \frac{dr}{dt} = -972\pi \text{ cm}^3\text{/sec}$$

9) A 10 ft ladder is leaning against a wall and sliding towards the floor. The foot of the ladder is sliding away from the base of the wall at a rate of 4 ft/sec. How fast is the top of the ladder sliding down the wall when the top of the ladder is 8 ft from the ground?

x = horizontal distance from base of ladder to wall y = vertical distance from top of ladder to floor t = time

Equation: $x^2 + y^2 = 10^2$ Given rate: $\frac{dx}{dt} = 4$ Find: $\left.\frac{dy}{dt}\right|_{y=8}$

$\left.\frac{dy}{dt}\right|_{y=8} = -\frac{x}{y} \cdot \frac{dx}{dt} = -3$ ft/sec, therefore: 3 ft/sec down the wall

10) An observer stands 2400 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 600 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 700 ft from the ground?

a = altitute of rocket z = distance from observer to rocket t = time

Equation: $a^2 + 5760000 = z^2$ Given rate: $\frac{da}{dt} = 600$ Find: $\left.\frac{dz}{dt}\right|_{a=700}$

$\left.\frac{dz}{dt}\right|_{a=700} = \frac{a}{z} \cdot \frac{da}{dt} = 168$ ft/sec

Evaluate each indefinite integral.

11) $\int (-18x^5 + 15x^4 + 15x^2)\, dx$

$-3x^6 + 3x^5 + 5x^3 + C$

12) $\int (10x^4 + 8x)\, dx$

$2x^5 + 4x^2 + C$

13) $\int (30x^5 + 15x^4 + 4x)\, dx$

$5x^6 + 3x^5 + 2x^2 + C$

14) $\int (8x - x^{-2} - 9x^{-4})\, dx$

$4x^2 + \frac{3}{x^3} + \frac{1}{x} + C$

15) $\int \left(12x^2 - \frac{12}{x^5}\right) dx$

$4x^3 + \frac{3}{x^4} + C$

16) $\int 6x^5 \, dx$

$x^6 + C$

17) $\int -\frac{10x^{\frac{1}{4}}}{4} \, dx$

$-2x^{\frac{5}{4}} + C$

18) $\int \left(10x^4 + \frac{25x^{\frac{2}{3}}}{3} + \frac{16x^{\frac{1}{3}}}{3}\right) dx$

$2x^5 + 5x^{\frac{5}{3}} + 4x^{\frac{4}{3}} + C$

19) $\int \frac{25\sqrt[4]{x}}{4} \, dx$

$5x^{\frac{5}{4}} + C$

20) $\int \left(25x^4 + 8x + 4\sqrt[3]{x}\right) dx$

$5x^5 + 4x^2 + 3x^{\frac{4}{3}} + C$

21) $\int 4x(6x^4 + 5x^3 + 2) \, dx$

$4x^6 + 4x^5 + 4x^2 + C$

22) $\int 2x^3(-3x^2 + 10x + 10) \, dx$

$-x^6 + 4x^5 + 5x^4 + C$

23) $\int \frac{-2x^2 - 2x - 9}{x^4} \, dx$

$\frac{2}{x} + \frac{1}{x^2} + \frac{3}{x^3} + C$

24) $\int \frac{-x^3 + 9x - 16}{x^5} \, dx$

$\frac{1}{x} - \frac{3}{x^3} + \frac{4}{x^4} + C$