ICM: Implicit Differentiation

Explicit: You can solve for y on one side of an equation

Implicit: you can't solve for y, so there are two variables to differentiate.

Before → $y = x^2$

$\frac{dy}{dx} x^2 = 2x$

After → $x^2 + y^2 = 1$

① Take the derivative w/ respect to x on both sides.

$2x + 2y\_\_\_ = 0$

↳ leave space because it's not an x term

② Multiply any y derivative by $\frac{dy}{dx}$.

$2x + 2y\frac{dy}{dx} = 0$

③ Use algebra to get any terms with $\frac{dy}{dx}$ on one side.

$2y\frac{dy}{dx} = -2x$

④ If necessary, factor out $\frac{dy}{dx}$. Then solve for $\frac{dy}{dx}$.

$\frac{2y\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

Ex. $x^2 - 2y^3 + 4y = 2$

$$2x - 6y^2 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$-2x \qquad\qquad -2x$$

$$-6y^2\frac{dy}{dx} + 4\frac{dy}{dx} = -2x$$

Factor out $\frac{dy}{dx}$.

$$\left.\begin{aligned}\frac{dy}{dx}(-6y^2+4) &= -2x\\ \frac{dy}{dx}2(-3y^2+2) &= -2x\end{aligned}\right\}\text{same}$$

Divide to solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{-2x}{2(-3y^2+2)} = \frac{-x}{(-3y^2+2)}$$

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Implicit Differentiation

We have been able to differentiate functions that are solved for y explicitly up to this point. Now we want to consider functions of the type $x^2 - 2y^3 + 4y = 2$. You can see that it would be quite challenging to solve for y as a function of x, explicitly.

Implicit Differentiation

- Realize differentiation is taking place with respect to x.
- When you differentiate terms involving x alone, you can differentiate as usual.
- When you differentiate terms involving y, you must apply the Chain Rule (because you are assuming that y is defined implicitly as a differentiable function of x.)

GUIDELINES FOR IMPLICIT DIFFERENTIATION

1. Differentiate both sides with respect to x.
2. Collect $\frac{dy}{dx}$ terms on one side = to all the other terms.
3. Factor out $\frac{dy}{dx}$
4. Solve for $\frac{dy}{dx}$

EX #1: Find $\frac{dy}{dx}$ for $x^2 - y^2 = 16$ at $(-5,3)$

EX #2: Find $\frac{dy}{dx}$ for $\boxed{xy} + y = 8$ at $(3,2)$

product rule

$x(1)\frac{dy}{dx} + y(1) + 1\frac{dy}{dx} = 0$

$-y \qquad -y$

$x\frac{dy}{dx} + \frac{dy}{dx} = -y$

$\frac{dy}{dx}\frac{(x+1)}{(x+1)} = \frac{-y}{(x+1)}$

$\frac{dy}{dx} = \frac{-y}{(x+1)}$ @$(3,2)$

$\frac{-2}{3+1} = \frac{-2}{4} = -\frac{1}{2}$

F1: 3X

F2: Y

EX #3: Find the instantaneous rate of change at $(1,1)$ for $x + 3xy - 2y^2 = 2$

$1 + 3x\frac{dy}{dx} + 3y - 4y\frac{dy}{dx} = 0$

$-1 \qquad -3y \qquad -1-3y$

$3x\frac{dy}{dx} - 4y\frac{dy}{dx} = -1-3y$

$\frac{dy}{dx}\frac{(3x-4y)}{(3x-4y)} = \frac{-1-3y}{(3x-4y)}$

@$(1,1)$ $\frac{-1-3(1)}{3(1)-4(1)} = \frac{-4}{-1} = \boxed{4}$

EX #4: Find $\frac{dy}{dx}$ given that $y^3 + 5y^2 - 5y - x^2 = -4$

EX #5: Find $\frac{dy}{dx}$ for $x + \sqrt{xy} = 6$ at $(3,3)$

EX #6: Find $\frac{dy}{dx}$ for $(x-y)^2 + y = 6$ at $(0,2)$

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