

Differentiate each function with respect to x .

1) $y = (-3x^2 + 5)^5$

$$\begin{aligned}\frac{dy}{dx} &= 5(-3x^2 + 5)^4 \cdot -6x \\ &= -30x(-3x^2 + 5)^4\end{aligned}$$

2) $y = (5x^2 + 3)^{\frac{1}{2}}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}(5x^2 + 3)^{-\frac{1}{2}} \cdot 10x \\ &= \frac{5x}{(5x^2 + 3)^{\frac{1}{2}}}\end{aligned}$$

3) $y = (-2x^4 + 5)^{-3}$

$$\begin{aligned}\frac{dy}{dx} &= -3(-2x^4 + 5)^{-4} \cdot -8x^3 \\ &= \frac{24x^3}{(-2x^4 + 5)^4}\end{aligned}$$

4) $y = \sqrt{4x^2 + 1}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}(4x^2 + 1)^{-\frac{1}{2}} \cdot 8x \\ &= \frac{4x}{(4x^2 + 1)^{\frac{1}{2}}}\end{aligned}$$

5) $f(x) = (3x - 1)(4x^5 + 3)^2$

$$\begin{aligned}f'(x) &= (3x - 1) \cdot 2(4x^5 + 3) \cdot 20x^4 + (4x^5 + 3)^2 \cdot 3 \\ &= (4x^5 + 3)(132x^5 - 40x^4 + 9)\end{aligned}$$

6) $y = \frac{(5x^3 + 1)^{-4}}{-x^5 + 4}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(-x^5 + 4) \cdot -4(5x^3 + 1)^{-5} \cdot 15x^2 - (5x^3 + 1)^{-4} \cdot -5x^4}{(-x^5 + 4)^2} \\ &= \frac{5x^2(17x^5 - 48 + x^2)}{(-x^5 + 4)^2 \cdot (5x^3 + 1)^5}\end{aligned}$$

7) $y = \left(\frac{-2x^5 - 3}{4x^3 - 1}\right)^2$

$$\begin{aligned}\frac{dy}{dx} &= 2 \cdot \frac{-2x^5 - 3}{4x^3 - 1} \cdot \frac{(4x^3 - 1) \cdot -10x^4 - (-2x^5 - 3) \cdot 12x^2}{(4x^3 - 1)^2} \\ &= \frac{4x^2(-2x^5 - 3)(-8x^5 + 5x^2 + 18)}{(4x^3 - 1)^3}\end{aligned}$$

$$8) y = ((5x^5 + 2)^4 + 4)^5$$

$$\begin{aligned} \frac{dy}{dx} &= 5((5x^5 + 2)^4 + 4)^4 \cdot 4(5x^5 + 2)^3 \cdot 25x^4 \\ &= 500x^4((5x^5 + 2)^4 + 4)^4 \cdot (5x^5 + 2)^3 \end{aligned}$$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

$$9) 1 = 3x^2 - 5y^3$$

$$\frac{dy}{dx} = \frac{2x}{5y^2}$$

$$10) 2x^2 + 2y^2 + 5y = 3$$

$$\frac{dy}{dx} = -\frac{4x}{4y + 5}$$

Write the equation of the tangent line through the given point.

$$11) 2 = 2x^3 - xy \text{ at } (-2, 9)$$

$$\left. \frac{dy}{dx} \right|_{\substack{x=-2 \\ y=9}} = -\frac{15}{2}$$

Differentiate each function with respect to x .

$$12) y = \sin 4x^3$$

$$\begin{aligned} \frac{dy}{dx} &= \cos 4x^3 \cdot 12x^2 \\ &= 12x^2 \cos 4x^3 \end{aligned}$$

$$13) y = (4\cos x)^{10}$$

$$(40\cos^9 x) * (-\sin x)$$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$ and the acceleration function $a(t)$.

14) $s(t) = -t^3 + 28t^2 - 196t$

$v(t) = -3t^2 + 56t - 196, a(t) = -6t + 56$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the times t when the particle changes directions.

15) $s(t) = t^3 - 22t^2 + 121t$

Changes direction at: $t = \left\{ \frac{11}{3}, 11 \right\}$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the intervals of time when the particle is moving left and moving right.

16) $s(t) = t^3 - 26t^2 + 169t$

Moving left: $\frac{13}{3} < t < 13$, Moving right: $0 \leq t < \frac{13}{3}, t > 13$

A particle moves along a horizontal line. Its velocity function is $v(t)$ for $t \geq 0$. For each problem, find the intervals of time when the particle is moving left and moving right.

17) $v(t) = 3t^2 - 16t$

Moving left: $0 < t < \frac{16}{3}$, Moving right: $t > \frac{16}{3}$

A particle moves along a horizontal line. Its velocity function is $v(t)$ for $t \geq 0$. For each problem, find the intervals of time when the particle is slowing down and speeding up.

18) $v(t) = 3t^2 - 60t + 225$

Slowing down: $0 \leq t < 5, 10 < t < 15$, Speeding up: $5 < t < 10, t > 15$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the displacement of the particle and the distance traveled by the particle over the given interval.

19) $s(t) = t^3 - 13t^2 + 40t; 0 \leq t \leq 6$

Displacement: -12

Distance traveled: 84