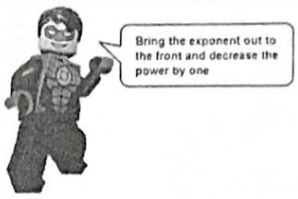
$\sqrt{x} = x^{1/2}$

AB Calculus: Rules for Differentiation **Name:** _____

Power Rule for Differentiation

If n is any number, then $\frac{d}{dx}[x^n] = n \cdot x^{n-1}$, provided x^{n-1} exists.



The key to using the power rule is to get comfortable using exponent rules to write a function as a power of x.

Example 1: Find the derivative of each of the following.

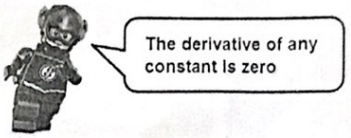
a) $f(x) = x^5$ $5x^4$

b) $f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$ $f' = \frac{2x^{-1/3}}{3} = \frac{2}{3\sqrt[3]{x}}$

c) $f(x) = \frac{1}{x^4} = x^{-4}$ $f' = -4x^{-5} = \frac{-4}{x^5}$

The Derivative of a Constant Function

If c is any constant value, then $\frac{d}{dx}[c] = 0$.

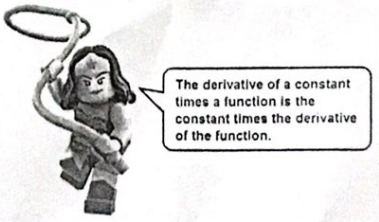


Example 2: Let $f(x) = 5$. Find $f'(x)$. $= 0$

The Constant Multiple Rule for Derivatives

If u is a differentiable function of x and c is a constant value, then

$$\frac{d}{dx}[cu] = c\frac{du}{dx}$$



Example 3: Find the derivative of each of the following.

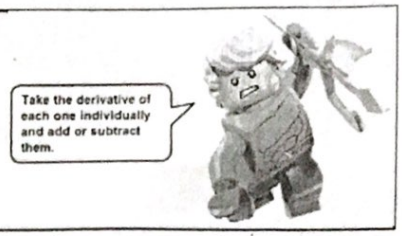
a) $f(x) = 5x^7$ $= 35x^6$

b) $f(x) = \frac{4}{5x^3}$ $= \frac{4}{5}x^{-3} = \frac{-12}{5}x^{-4} = \frac{-12}{5x^4}$

The Sum and Difference Rule for Derivatives

If u and v are differentiable functions of x, then wherever u and v are differentiable

$$\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$$



Example 3: Find the derivative of each of the following.

a) $f(x) = x^3 + 4x^2 - 2x + 7$

$3x^2 + 8x - 2$

$-2x$

b) $f(x) = \frac{3}{(-2x)^4} - \frac{x}{2} + \frac{1}{4}$

$\frac{3x^{-4}}{16} - \frac{1}{2}x + \frac{1}{4}$

$f' = \frac{-3}{4x^5} - \frac{1}{2}$

PRACTICE

1. What does the derivative of a function tell you about the function?

2. What is the power rule for derivatives? (How do you take the derivative of $y = x^n$?

3. For each of the following, find $\frac{dy}{dx}$

$-5x^{-2} + 6x^{-1}$

a) $y = -2x^3 + x$

$-6x^2 + 1$

b) $y = \frac{x^4}{3} - \frac{x^2}{7} + 5$

$\frac{4}{3}x^3 - \frac{2}{7}x$

c) $y = -\frac{5}{x^2} + \frac{6}{x} - 8x^3$

$\frac{10}{x^3} - \frac{6}{x^2} - 24x^2$

d) $y = \frac{x^{-3}}{2} + 5x^{-4} - 3x^{-6}$

$\frac{-3}{2x^4} - \frac{20}{x^5} + \frac{18}{x^7}$

e) $y = 5x^4 + 2x^3 - 8x^2 - 7x + 11$

$20x^3 + 6x^2 - 16x - 7$

f) $y = 7x - 8$

7

Foil

g) $y = (x^2 - 3)(x + 4)$

$x^3 + 4x^2 - 3x - 12$

$3x^2 + 8x - 3$

$y = \sqrt{x} + \frac{3}{\sqrt{x}} - 6x^{\frac{5}{3}} + \frac{7}{x^3}$

$x^{1/2} + 3x^{-1/2} - 6x^{5/3} + 7x^{-3}$

$\frac{1}{2\sqrt{x}} - \frac{3}{2\sqrt{x^3}} - 10\sqrt[3]{x^2} - \frac{21}{x^4}$

i) $y = \frac{x^5 - 2x^4 + 3x^3}{x^5}$

$1 - 2x^{-1} + 3x^{-2}$

$\frac{2}{x^2} - \frac{6}{x^3}$