Solving Exponential & Log Equations

ex. $3^x = 243$

Roll out!

$\log_3 243 = x$

Change of Base: $\frac{\log 243}{\log 3}$

$x = 5$

ex. $(2^{3x})(2^{5x}) = 16$

* use exponent rules

$2^{8x} = 16$

Roll out!

$\log_2 16 = 8x$

$x = .5$

ex. $\left(\frac{64}{125}\right)^{2x-8} = \left(\frac{25}{16}\right)^x$

$\left(\frac{4^3}{5^3}\right)^{2x-8} = \left(\frac{5^2}{4^2}\right)^x$

change bases

$\left(\frac{4}{5}\right)^{3(2x-8)} = \left(\frac{5}{4}\right)^{2x}$

use exponent rules backwards

$\left(\frac{4}{5}\right)^{3(2x-8)} = \left(\frac{4}{5}\right)^{-2x}$

flip to get same base on both sides, but make the power negative!

$3(2x-8) = -2x$

$6x - 24 = -2x$

$-6x \qquad -6x$

$-24 = -8x$

$x = 3$

ex. $\log_3(2x-1) = 3$ Roll out!

$3^3 = 2x-1$

$27 = 2x-1$

$28 = 2x$

$x = 14$

ex. $\log_7(3x-11) = \log_7(x-3)$ one log base 7 = one log base 7 drop the logs!

$3x-11 = x-3$

$2x-11 = -3$

$2x = 8$

$x = 4$

ex. $\log(x+5) - \log(x-1) = \log(x+2) - \log(x-3)$ use log rules

$\log\left(\frac{x+5}{x-1}\right) = \log\left(\frac{x+2}{x-3}\right)$ one log base 10 = one log base 10

$\frac{x+5}{x-1} = \frac{x+2}{x-3}$ drop logs!

$(x+5)(x-3) = (x+2)(x-1)$ cross multiply FOIL

$x^2-3x+5x-15 = x^2-x+2x-2$

$x^2+2x-15 = x^2+x-2$

$x-15 = -2$

$x = 13$

ex. $4\ln_e x - \ln_e 2 = \ln_e 128$

$\ln_e\left(\frac{x^4}{2}\right) = \ln_e 128$ $\frac{x^4}{2} = 128$ $x^4 = 256$

$x = \sqrt[4]{256} = \{4, -4\}$ ↑ extraneous