Graph Theory:
Directed Graphs

☆ Some relationships do not go in both directions. Consider Ann and Kathy. If Ann is the mother of Kathy, then Kathy can not also be the mother of Ann.

☆ When an edge has direction it is called a **directed edge**.

☆ A graph with all directed edges is called a **directed graph**.

☆ **Example: Modeling the Spread of Rumors:**
There is a group of 4 friends- Buffy, Cordelia, Willow, and Xander. These girls often share stories and secrets. The following information has been collected about the group:

- If Buffy hears a rumor, she will share it with Cordelia, but Cordelia will not tell Buffy a rumor that she hears.
- Cordelia and Xander will tell any rumors that they hear to each other.
- Cordelia tells Willow rumors she hears, but Willow does not relay rumors to Cordelia.

Create a model to show how rumors are spread among the 4 friends.

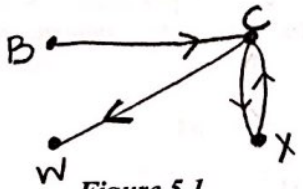


Figure 5.1

☆ If X and Y are vertices of a directed graph and it is possible to start at vertex X and follow a sequence of edges in the directions indicated, then this would be an example of a **directed path** from X to Y.

☆ The *length* of a directed path is the number of directed edges along that path.

☆ **Example:** Use the directed graph shown in Figure 5.2 to answer the following questions. (Draw arrows as I have done below to make it a directed graph, first...oops!)

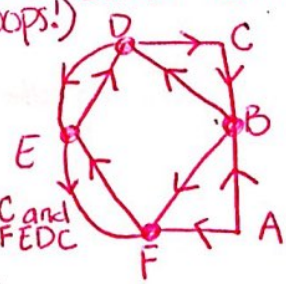


Figure 5.2

1. Can you find 2 directed paths from A to C? yes! ABFEDC and AFEDC
2. Are there any directed paths from C to A? no!
3. What is the length of the directed path [redacted]? ABFEDC?

5

☆ **Example: Modeling Influence**

A group of parents wants the town council to install a traffic light at a dangerous intersection. They feel that their best strategy is to obtain the support of the most influential member of the council.

Council Member	Influences these people
Alvarez	Cohen, Ellis, Ferraro
Baker	Ferraro
Cohen	Baker, Ellis
Davis	Baker, Cohen, Ellis
Ellis	Baker
Ferraro	Cohen, Ellis

Model the council members' influence with a directed graph.

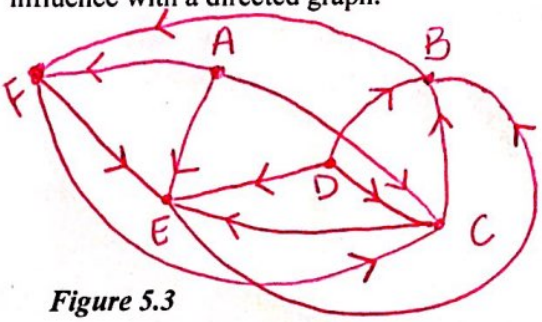


Figure 5.3

☆ Who appears to be the most influential?

A + D since they both influence 3 people

☆ Often, it is more important to look at deeper influences. We will investigate the two-stage influence of the city council members.

EX: consider A to B. A does not directly influence B, but if he influences C, then C might influence B (based on the graph or chart). Also, if A influences E, then E might also influence B based on the graph/chart. So A actually has 2 chances to influence B. Place a 2 in the box!

To

From

	A	B	C	D	E	F
A	0	2	2	0	3	1
B	0	0	1	0	1	1
C	0	2	0	0	1	1
D	0	3	1	0	2	1
E	0	1	0	0	0	1
F	0	2	1	0	2	0

EX: consider A to C. A directly influences C (based on the graph/chart). Also, if A influences F, then F might influence C (based on the graph/chart). So that's 2 total influences. Put a 2 in the box!

☆ To determine the most influential person using the **one-and-two stage influences**, we must find the council member who has the most ways to influence others.

☆ Rank the council members according to one-and-two stage influence:

Add the rows!

A = 0 + 2 + 2 + 0 + 3 + 1 = 8

B = 0 + 0 + 1 + 0 + 1 + 1 = 3

C = 4

D = 7

E = 2

F = 5

Most Influencial: Alvarez

Rankings:

A
D
F
C
B
E

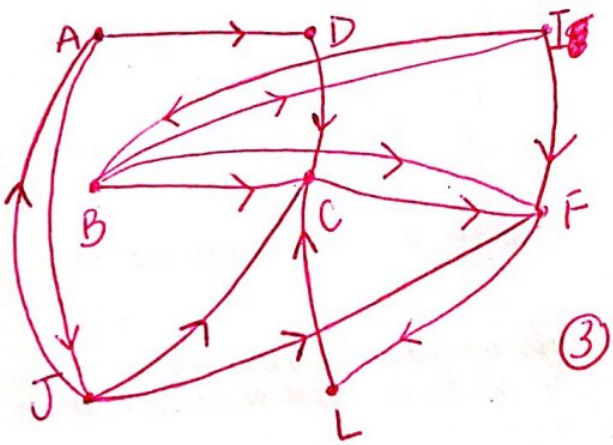
☆ **Example: Modeling the Spread of a Disease**

In a small town in southwestern New Mexico, eight people have been reported to be suffering from a disease known as the haunta virus. Officials have isolated these individuals and hope that no others have contracted the virus. The officials believe that the virus was introduced by one person in the town and then spread to others. Use the information in the table provided to determine whether or not all of the infected people in the town have been isolated.

Patient	Could have passed disease onto:
Amanda	Dustin, Jackson
Brian	Caterina, Frank, Ina
Caterina	Frank
Dustin	Caterina
Frank	Louisa
Ina	Brian, Frank
Jackson	Amanda, Caterina, Frank
Louisa	Caterina

☆ Have the officials isolated everyone in the town that has the virus?

① Make a directed graph.



② If one of these people started it, there would be a path from that person to every other person...

③ Look at A + J... how did they get the disease??
It had to be someone not on this graph, because no one else connects to them.
We also know A or J did not start the virus because their path doesn't get to everyone else...

④ So no... all infected people are not isolated because someone not on this graph started it... someone else is out there!