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1.2 Finding Limits Graphically and Numerically (Part I)

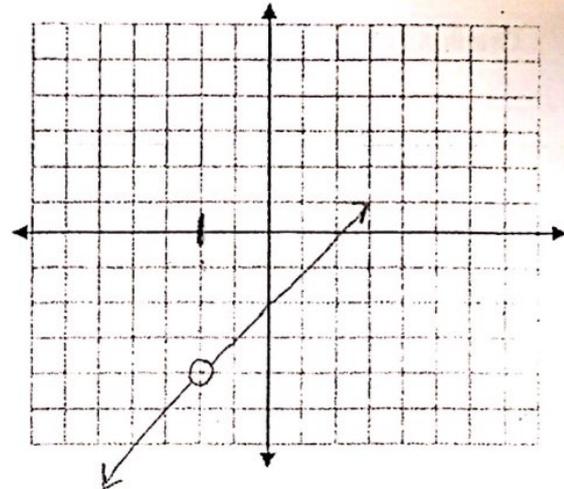
Goal: Find the limit of a function both graphically and numerically of a relation.

Limit: The value that y approaches as the value of x gets closer to a given number.

0/0 implies it is a Removable Discontinuity—we call 0/0 Indeterminate.

Graphically

Graph $f(x) = \frac{x^2 - 4}{x + 2}; x \neq -2$



What value does y approach, as x gets closer to -2 ?

Numerically

Using the function $f(x) = \frac{x^2 - 4}{x + 2}$, complete the tables.

$x \rightarrow -2^-$ { x is approaching -2 from the left}

x	-3	-2.5	-2.1	-2.01	-2.001
$f(x)$	-5	-4.5	-4.1	-4.01	-4.001

$x \rightarrow -2^+$ { x is approaching -2 from the right}

x	-1	-1.5	-1.9	-1.99	-1.999
$f(x)$	-3	-3.5	-3.9	-3.99	-3.999

$x \rightarrow -2^- \quad f(x) \rightarrow -4$
 $x \rightarrow -2^+ \quad f(x) \rightarrow -4$

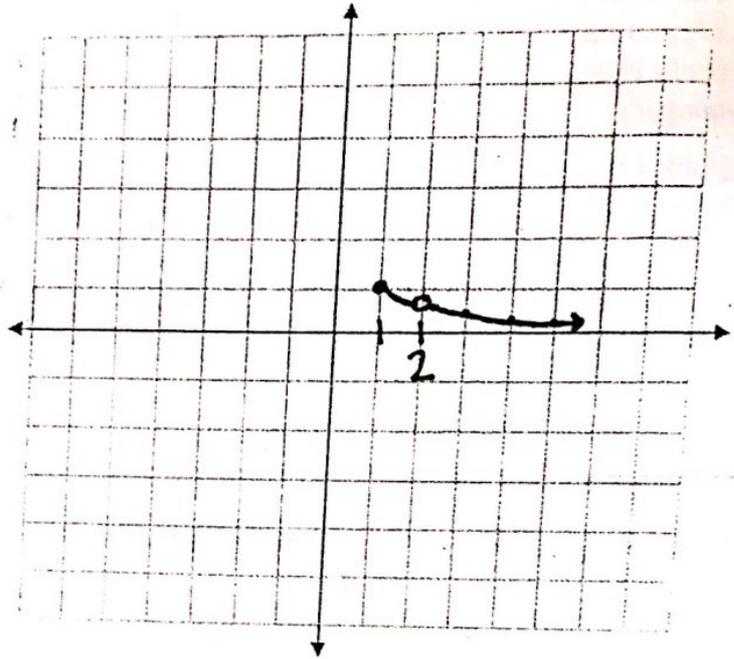
Each of these is referred to as a one-sided limit.

$\lim_{x \rightarrow -2} f(x) = -4$

{This is read as: The limit as x approaches -2 is -4}

2/18 Finding Limits Graphically and Numerically—continued.

Graph $f(x) = \frac{\sqrt{x-1}-1}{x-2}; x \neq 2$



$x \rightarrow 2^-$ {x is approaching 2 from the left}

x	1	1.5	1.9	1.99	1.999
f(x)	1	.5858	.5132	.5013	.5001

$x \rightarrow 2^+$ {x is approaching 2 from the right}

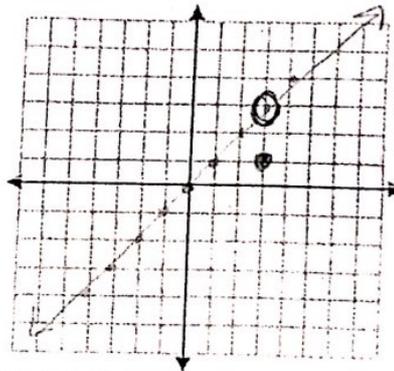
x	3	2.5	2.1	2.01	2.001
f(x)	.4142	.4495	.4881	.4988	.4999

$\lim_{x \rightarrow 2^-} f(x) = \frac{1}{2} \left\{ \begin{array}{l} f(x) \rightarrow .5 \end{array} \right.$
 $\lim_{x \rightarrow 2^+} f(x) = \frac{1}{2} \left\{ \begin{array}{l} f(x) \rightarrow .5 \end{array} \right.$

$\lim_{x \rightarrow 2} f(x) = .5$

{The limit as x approaches 2 is 1/2}

Look at $f(x) = \begin{cases} x, & x \neq 3 \\ 1, & x = 3 \end{cases}$ linear $\lim_{x \rightarrow 3} f(x) = 3$



1.2 Part I Pg. 54: 1, 3, 7

(2)

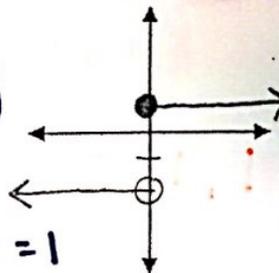
1.2 Limits That Do Not Exist (Part II)

Goal: To be able to recognize and illustrate the three ways a limit can fail.

1. $f(x)$ approaches a different number from the left and right.

$$f(x) = \begin{cases} -2 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

jump discontinuity



$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 0^-} f(x) = -2$$

$$\lim_{x \rightarrow 0^+} f(x) = 1$$

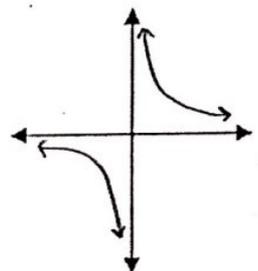
2. $f(x)$ increases or decreases without bound as x approaches a given value.

$$f(x) = \frac{1}{x}$$

$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 0^-} f(x) = -\infty$$

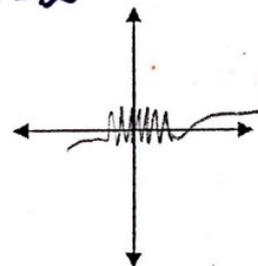
$$\lim_{x \rightarrow 0^+} f(x) = \infty$$



3. $f(x)$ oscillates between two fixed values as x approaches a given value.

$$f(x) = \sin \frac{1}{x}$$

$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$



The graph to the right represents the function $f(x)$. Find each of the following:

2/19 $f(1) = 2$

$f(5) = 2$

$f(6) = 3$

$f(6.5) = \text{und.}$

$f(-1) = .3$

$f(7) = 3$

$f(-5) = \text{und.}$

$f(0) = -.3$

$\lim_{x \rightarrow 1} f(x) = \text{DNE}$

$\lim_{x \rightarrow 5} f(x) = 5$

$\lim_{x \rightarrow 6} f(x) = \text{DNE}$

$\lim_{x \rightarrow 6.5} f(x) = \text{DNE}$

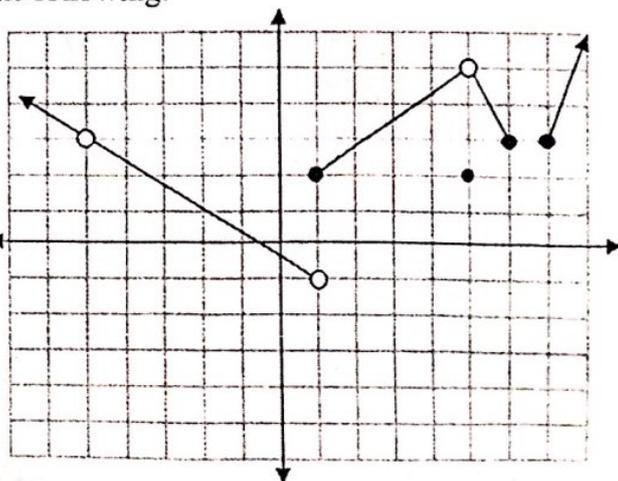
$\lim_{x \rightarrow -1} f(x) = .3$

$\lim_{x \rightarrow 7} f(x) = \text{DNE}$

$\lim_{x \rightarrow -5} f(x) = 3$

$\lim_{x \rightarrow 0} f(x) = -.3$

$\lim_{x \rightarrow 1} f(x) = \text{DNE}$

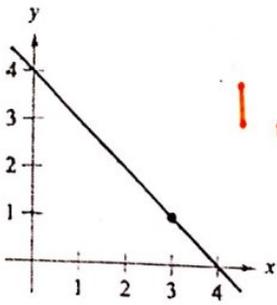


L, R, overall



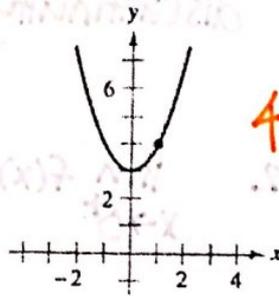
Use the graph to find the limits.

1. $\lim_{x \rightarrow 3} (4 - x)$



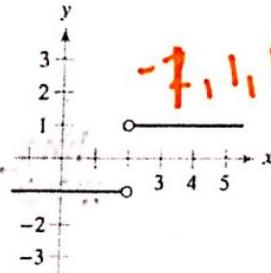
1, 1, 1

2. $\lim_{x \rightarrow 1} (x^2 + 3)$



4, 4, 4

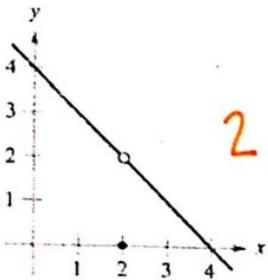
3. $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$



-1, 1, DNE

4. $\lim_{x \rightarrow 2} f(x)$

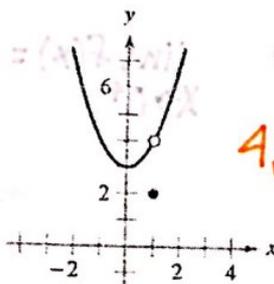
$$f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$$



2, 2, 2

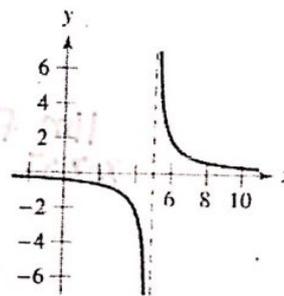
5. $\lim_{x \rightarrow 1} f(x)$

$$f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$



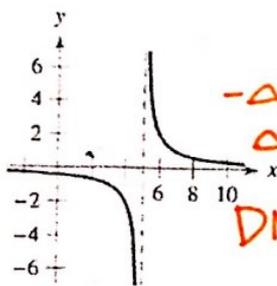
4, 4, 4

6. $\lim_{x \rightarrow 5} \frac{2}{x - 5}$



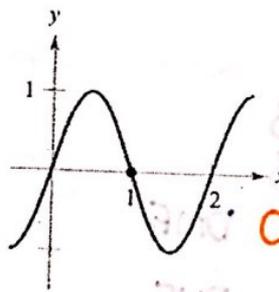
-\infty, \infty, DNE

7. $\lim_{x \rightarrow 5} \frac{2}{x - 5}$



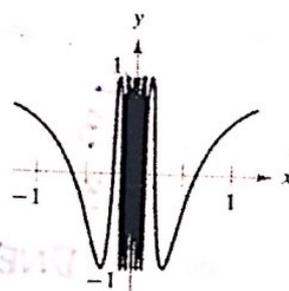
-\infty, \infty, DNE

8. $\lim_{x \rightarrow 1} \sin \pi x$



0, 0, 0

9. $\lim_{x \rightarrow 0} \cos \frac{1}{x}$



DNE

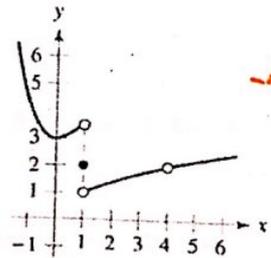
Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not explain why.

10. (a) $f(1)$ **2**

(b) $\lim_{x \rightarrow 1} f(x)$ **DNE**

(c) $f(4)$ **und.**

(d) $\lim_{x \rightarrow 4} f(x)$ **2, 2, 2**



11. (a) $f(-2)$ **und**

(b) $\lim_{x \rightarrow -2} f(x)$ **DNE**

(c) $f(0)$ **4**

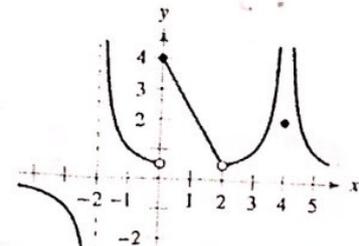
(d) $\lim_{x \rightarrow 0} f(x)$ **DNE**

(e) $f(2)$ **und.**

(f) $\lim_{x \rightarrow 2} f(x)$ **2, 1/2, 1/2, 1/2**

(g) $f(4)$ **2, 1/2, 1/2, 1/2**

(h) $\lim_{x \rightarrow 4} f(x)$ **DNE**



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