RELATED RATES NOTES

Given the following equation: $y^4 - 2y^3 - y^2 + 2y = x^3 - 3x^2 + 2x$

1. Find dy/dx $(4y^3 - 6y^2 - 2y + 2)\frac{dy}{dx} = 3x^2 - 6x + 2$

$= \frac{3x^2 - 6x + 2}{4y^3 - 6y^2 - 2y + 2}$

2. Find the exact x-coordinates of the horizontal tangent(s)

3. Find the exact y-coordinates of the vertical tangent(s)

4. Find the equation of the tangent line at the points (0, 1)

$m = 2/-2 = -1$ $\quad y - 1 = -1(x - 0)$ $\quad y - 1 = -x$ $\quad \boxed{y = -x + 1}$

Example #1

A particle moves along the curve $y = 3x^2 - 6x$ so that the rate of change of the x-coordinate dx/dt is 2 units/sec. Find the rate of change of the y-coordinate, dy/dt, when the particle is at the origin. (0,0)

Take the derivative of both sides with respect to t	$1\frac{dy}{dt} = 6x\frac{dx}{dt} - 6\frac{dx}{dt}$
Substitute in the known info	$\frac{dy}{dt} = 6(0)(2) - 6(2)$
Solve for the indicated variable	$\frac{dy}{dt} = -12$ units/sec

A strategy for Solving Related Rates Problems

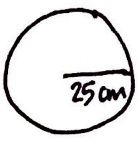
Step 1	Read the problem carefully.
Step 2	Draw a diagram if possible
Step 3	Introduce notation. Assign symbols to all quantities that are functions of time
Step 4	Express the given information and the required rate in terms of derivatives
Step 5	Write an equation that relates the various quantities of the problem. If necessary, use Geometry to eliminate one of the variables by substitution
Step 6	Use the Chain Rule to differentiate both sides of the equation with respect to t
Step 7	Substitute the given information in the resulting equation and solve for the unknown rate.

Example 2.

Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm³/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm?

$V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 100\ cm^3/s$

$\frac{dr}{dt} = ?$



$r = 25\ cm$

$1\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$100 = 4\pi(25)^2 \frac{dr}{dt} \rightarrow \frac{100}{4\pi(25)^2} = \boxed{\frac{dr}{dt} = \frac{1}{25\pi}\ cm/s}$

Example 3:

$A = \pi r^2$ $\quad dr/dt = 2\ ft/s$

Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2ft/sec. How fast is the area of the spill increasing when the radius of the spill if 60ft?

$\frac{dA}{dt} = ?$ $\quad r = 60\ ft.$

$1\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \rightarrow \frac{dA}{dt} = 2\pi(60)(2) = \boxed{240\pi\ ft^2/s}$

Example 4

The length of *l* of a rectangle is decreasing at the rate of 2 cm/sec while the width *w* is increasing at the rate of 2 cm/sec, When *l* = 12 and *w* = 5, find the rates of change of

$\frac{dL}{dt} = -2\ cm/s$ $\quad \frac{dw}{dt} = 2\ cm/s$ $\quad L = 12\ cm$ $\quad W = 5\ cm$

a) The area

$A = L \cdot W$

$\frac{dA}{dt} = L\,dw/dt + w\,dL/dt \;:\; \frac{dA}{dt} = 12(2) + (5)(-2) = \boxed{14\ cm^2/s}$

b) The perimeter

$P = 2L + 2W$

$\frac{dP}{dt} = 2\,dL/dt + 2\,dw/dt \quad \frac{dP}{dt} = 2(-2) + 2(2) = \boxed{0\ cm/s}$

c) The length of the diagonal of the rectangle

Example 5

A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

a) How fast is the top of the ladder sliding down the wall at that moment?

b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?

c) At what rate is the angle between the ladder and the ground changing at that moment?