<u>Related Rates</u>

The derivative, $\frac{dy}{dx}$, of a function, $y = f(x)$, is its instantaneous rate of change with respect to the variable x.

- When a function describes either position or distance, its rate of change is interpreted as velocity.
- In general, a time rate of change answers the question: *How fast is a quantity changing?*
  - For example, if V is volume that is changing in time then $\frac{dV}{dt}$ is the rate, or how fast, the volume is changing with respect to time.
  - If a person is walking toward a street lamp at a constant rate of 3 feet per second, then we know that the distance is decreasing, so $\frac{dx}{dt} = -3\,\frac{ft}{sec}$.
  - If they walk away from the lamp then the distance is increasing and the rate of change becomes positive or $\frac{dx}{dt} = 3\,\frac{ft}{sec}$.

GUIDELINES FOR SOLVING RELATED RATE PROBLEMS:

1. Make a sketch and label the quantities.
2. Read the problem and identify all quantities as: "FIND", "WHEN", and "GIVEN" with the appropriate information.
3. Write an equation involving the variables whose rates of change either are given or are to be determined.
4. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time, t.*
5. *AFTER COMPLETING STEP 4,* substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

EX #1: Suppose ~~x and y~~ are both differentiable functions of t and are related by the equation $y = x^2 - 3x$. Find $\frac{dy}{dt}$ when $x = 3$ given that $\frac{dx}{dt} = 2$, when $x = 3$.

$1 = 2x - 3$

$1\frac{dy}{dt} = 2x\frac{dx}{dt} - 3\frac{dx}{dt}$

$\frac{dy}{dt} = 2\cdot 3(2) - 3(2)$

$\frac{dy}{dt} = 6$ units/time

Jean Adams

EX. #2: A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

1. KNOW: $A = \pi r^2$

2. GIVEN: $\frac{dr}{dt} = 1$ ft/s $\quad r = 4$ ft.

3. FIND: $\frac{dA}{dt} = ?$

$A = \pi r^2$

① $1\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

② $\frac{dA}{dt} = 2\pi(4)(1) = \boxed{8\pi \text{ ft}^2/\text{s}}$

EX. #3: Air is being pumped into a spherical balloon at a rate of 800 cubic centimeters per minute. How fast is the radius of the balloon changing at the instant the radius is 30 centimeters?

1. KNOW: $V = \frac{4}{3}\pi r^3$

2. GIVEN: $\frac{dV}{dt} = 800$ cm³/min $\quad r = 30$ cm

3. FIND: $\frac{dr}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$1\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\frac{800}{4\pi(30)^2} = \frac{4\pi(30)^2}{4\pi(30)^2}\frac{dr}{dt}$

$\frac{800}{4\pi 30^2}$ cm/min $= \frac{dr}{dt} = \boxed{\frac{2}{9\pi} \text{ cm/min}}$

Jean Adams