ICM Day 3: Solving Polynomials

Synthetic Division

Ex. Divide $3x^3 - 2x^2 + 3x - 4$ by $(x-3)$.

↗ this is a factor!

$x - 3 = 0$
$+3 \quad +3$
$x = 3$
↳ in the house!

3	3	−2	3	−4
	↓	9	21	72
	3	7	24	68

$3x^2 + 7x + 24 + \frac{68}{x-3}$ ← Remainder!

Ex. Test to see if 4 is a root of $x^3 - 12x^2 + 48x - 64$.

4	1	−12	48	−64
	↓	4	−32	64
4	1	−8	16	0
	↓	4	−16	
4	1	−4	0	
	↓	4		
	1	0		

→ No Remainder, 4 is a root! Is it a double root?

→ No Remainder, 4 is a double root! Is it a triple root?

→ No Remainder, 4 is a triple root!

Rational Root Theorem

- a way to list POSSIBLE rational roots
- divide factors of constant term by factors of leading coefficient to get possible roots
- use syn div to test

ex. Find the roots (all roots) of $x^3 - 2x^2 - x + 2$.

factors of constant (2) : $\pm 1, \pm 2$
factors of leading coeff (1) : $\frac{1, 2}{1} = \pm 1, \pm 2$

POSSIBLE roots

Pick a possible root to test -

2 | 1 -2 -1 2
↓ 2 0 -2
1 0 -1 : 0 → No Remainder, 2 is a root!

$x^2 - 1$ is the new polynomial.

Factor!

$(x+1)(x-1)$ Difference of Perf. Sq.

$x = -1$ $x = 1$

Answer: $x = 2, 1, -1$