Rational Root Theorem

- A way to list possible rational roots. ← can be a fraction
- Divide the factors of the constant term (p) by the factors of the leading coefficient (q).
- Use Synthetic Division to test the possible roots.

Ex: Find all roots of $x^3 - 2x^2 - x + 2$. (q, p)

Factors of p: $\pm 2, \pm 1$
Factors of q: ± 1

possible roots: $\pm 2, \pm 1$

1] 1 -2 -1 2
↓ 1 -1 -2
1 -1 -2 0 ✓

2] 1 -1 -2 0
↓ 2 2
1 1 0 ✓

$X+1$

$X = 1, 2, -1$ ← roots
$(X-1)(X-2)(X+1)$
↑ factors

Ex: Find all roots of $3x^3 - 11x^2 + 5x + 3$.

Factors of p: $\pm 3, \pm 1$
Factors of q: $\pm 3, \pm 1$

possible Roots: $\pm 3, \pm 1, \pm \frac{1}{3}$

1] 3 -11 5 3
↓ 3 -8 -3
3 -8 -3 0 ✓

$3x^2 - 8x - 3$

$(3X+1)(X-3)$

$X = -1/3, 3, 1$

$(3X+1)(X-3)(X-1)$

Synthetic Division

Why is it useful?

Determines if something is a root (rem. of zero)

Ex: Divide $3x^3 - 2x^2 + 3x - 4$ by $x - 3$ using synthetic division.

factor

$$\begin{array}{r|rrrr} 3 & 3 & -2 & 3 & -4 \\ & \downarrow & 9 & 21 & 72 \\ \hline & 3 & 7 & 24 & 68 \end{array}$$

remainder

$$3x^2 + 7x + 24 + \frac{68}{x-3}$$

Ex: Use synthetic division to determine if $x - 4$ is a factor of $-2x^5 + 6x^4 + 10x^3 - 6x^2 - 9x + 4$.

$$\begin{array}{r|rrrrrr} 4 & -2 & 6 & 10 & -6 & -9 & 4 \\ & \downarrow & -8 & -8 & 8 & 8 & -4 \\ \hline & -2 & -2 & 2 & 2 & -1 & 0 \end{array}$$

yes!

Ex: Use synthetic division to determine how many times 4 is a root of $x^3 - 12x^2 + 48x - 64$.

$$\begin{array}{r|rrrr} 4 & 1 & -12 & 48 & -64 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$