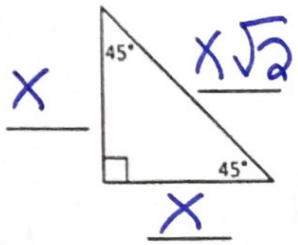
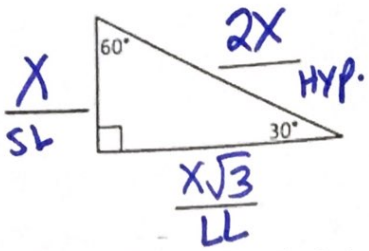
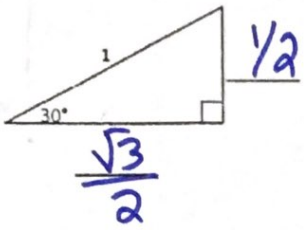
Unit Circle

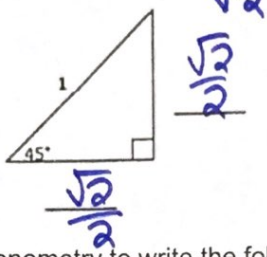
We're going to be using special right triangles, so first, refresh your memory on the rules.



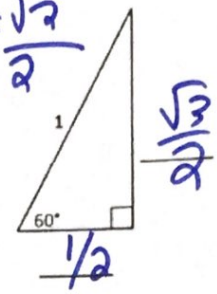


Use special right triangles to fill in the missing side lengths below. NO DECIMALS!





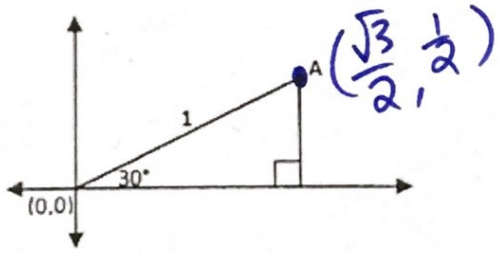
$\frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



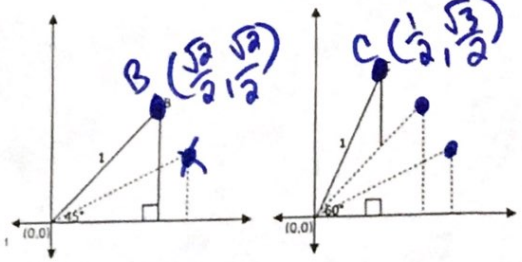
Now use what you know about trigonometry to write the following sine and cosine ratios for the angles shown in the triangles above. Simplify!

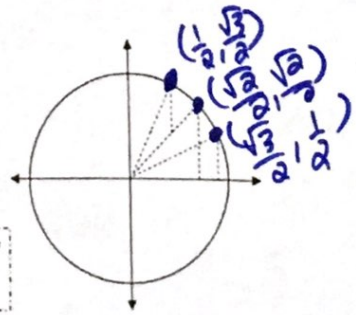
sin 30 = $\frac{1}{2}$ sin 45 = $\frac{\sqrt{2}}{2}$ sin 60 = $\frac{\sqrt{3}}{2}$ cos 30 = $\frac{\sqrt{3}}{2}$ cos 45 = $\frac{\sqrt{2}}{2}$ cos 60 = $\frac{1}{2}$

Let's take one of those triangles and put it on a coordinate plane. What would the x- and y- coordinates of Point A (formed by a 30 degree angle) be? (Hint: Use the lengths of the sides of the triangle.)



Do the same for a 45 and 60 degree angle. If you look at the three points we've traced out so far, they all lie on a circle centered at the origin.





What is the radius of this circle?

1

Tangent = $\frac{\sin}{\cos} = \frac{y}{x}$

Use what you've done with triangles so far to fill in the following table:

Angle Measure	Coordinate on Unit Circle	Value of Sine (y)	Value of Cosine (x)
30°	$(\sqrt{3}/2, 1/2)$	$1/2$	$\sqrt{3}/2$
45°	$(\sqrt{2}/2, \sqrt{2}/2)$	$\sqrt{2}/2$	$\sqrt{2}/2$
60°	$(1/2, \sqrt{3}/2)$	$\sqrt{3}/2$	$1/2$

The other three trigonometric functions are cosecant, secant, and cotangent. Write their ratios below:

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

Before we begin using the unit circle, we need to understand the relationship between degrees and radians.

Rule for converting from Degrees to Radians: $\frac{\pi}{180} = \frac{\quad}{\quad}$

Rule for converting from Radians to Degrees: $\frac{-220\pi}{180}$

Practice

Convert the measurements below to either radians (#1 - 3) or degrees (#4 - 6).

1. 30 degrees (CCW) $\frac{\pi}{180} = \frac{\pi}{30}$ $\frac{\pi}{6}$

2. -220 degrees (CW) $\frac{\pi}{180} = \frac{\pi}{-220}$ $-11\pi/9$

3. 400 degrees $\frac{\pi}{180} = \frac{\pi}{400}$ $\frac{20\pi}{9}$

4. $\frac{4\pi}{3}$ radians $\frac{\pi}{180} = \frac{4\pi/3}{}$ $240°$

5. $\frac{-5\pi}{36}$ radians $\frac{\pi}{180} = \frac{-5\pi/36}{}$ $-25°$

6. $\frac{7\pi}{4}$ radians $\frac{\pi}{180} = \frac{7\pi/4}{}$ $315°$

Above, we converted an angle of 400 degrees to radians. How can there be a 400 degree angle if a circle is a total of 360 degrees?

Definition of Coterminal Angles: Angles that end in the same spot on the unit circle.

Practice

Find one positive and one negative coterminal angle for each of the following.

1. 65 degrees $425°$, $-295°$

2. 540 degrees $-180°, 180°$

3. $\frac{13\pi}{18}$ radians $\frac{13\pi}{18} + \frac{36\pi}{18}$ $\frac{49\pi}{18}, \frac{-23\pi}{18}$

4. $\frac{14\pi}{9}$ radians $\frac{14\pi}{9} + \frac{18\pi}{9}$ $\frac{32\pi}{9}, \frac{-4\pi}{9}$